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How do contrasting cases and self-explanation promote learning? Evidence from fraction division☆

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ABSTRACT

Past research has shown that both contrasting cases instruction and prompts to self-explain promote students' learning in mathematics. However, it is not clear whether these instructional approaches enhance learning through similar mechanisms or whether each supports learning in distinct ways. The purpose of this study was to investigate the unique and combined effects of comparison, defined as noticing similarities and differences, and explanation, defined as making sense of problems, on student learning, and to assess whether these processes are more effective when combined than when implemented separately. We also investigated potential mechanisms involved in comparison and explanation. We addressed these issues in the domain of fraction division. Prompts to self-explain promoted conceptual learning, but inviting comparison, without prompts to self-explain, did not. The quality of students' self-explanations was affected by both prompts to self-explain and encouragement to compare. The findings suggest that contrasting cases instruction is effective, at least in part, because it typically involves self-explanation.

1. Introduction

Comparing problems or solution methods is often recommended as an element of effective mathematics instruction (NCTM, 2000). Noticing similarities and differences between problems has been hypothesized to foster active reflection and connecting ideas (e.g., Ball, 1993; Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005).

One instructional method that involves comparison is *contrasting cases* instruction. In contrasting cases instruction, analogous problems are purposely juxtaposed and explicitly compared. In practice, however, contrasting cases instruction typically involves both *comparing* and *explaining* the meaning or importance of those similarities and differences. The purpose of this research is to explore how these two factors—comparison and explanation—influence learning, both separately and in combination.

1.1. Comparison

Contrasting cases instruction supports learning better than instruction that involves studying a single example or studying two examples separately. In one study, 7th-grade students who compared two strategies for solving equations were more successful at solving transfer problems, more likely to explore alternative strategies, and more likely to use a demonstrated shortcut than were students who studied the two strategies sequentially (Rittle-Johnson & Star, 2007). Thus, contrasting cases instruction fostered deeper learning.

Contrasting cases instruction typically involves both *comparison*, which we define as noticing similarities and differences between problems, and also some form of student-led *explanation*. Students are typically encouraged to "make sense" of the cases and to consider or discuss their similarities and differences (e.g., Rittle-Johnson, Star, & Durkin, 2012; Schwartz, Chase, Oppezzo, & Chin, 2011). This aspect of contrasting cases instruction bears a strong resemblance to another instructional approach that promotes students' understanding of mathematics: self-explanation (e.g., Chi, Bassok, Lewis, Reimann, & Glaser, 1989). In light of this similarity, it seems possible that the benefits of contrasting cases instruction could be due to students comparing the problems, to their

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explaining the problems, or to some combination of both. To date, little is known about the role of explanation in contrasting cases instruction.

1.2. Explanation

Many studies have documented the value of producing explanations for learning (e.g., Aleven & Koedinger, 2002; Chi, de Leeuw, Chiu, & LaVancher, 1994). Much of this research has focused on *self-explanations*, which are self-directed explanations that learners generate in making sense of new information. In a seminal study of college students learning from worked physics problems, Chi et al. (1989) found that students who were better problem-solvers at posttest engaged in deeper self-explanation during study. This deep self-explanation included several types of activities: monitoring understanding, making connections to prior knowledge, and making inferences so as to better generalize from examples (Chi et al., 1989; Chi & VanLehn, 1991).

Many subsequent studies have shown that prompting students to self-explain while studying worked examples fosters learning. Across studies, a range of prompts have been used to elicit self-explanations, including prompts to explain material to oneself, to explain why a solution is correct or incorrect, to make connections (e.g., to prior knowledge), or to explain how an example is an instance of a broader category. Studies vary in terms of whether they include self-explanation training (e.g., Bielaczyc, Pirolli, & Brown, 1995) and whether participants are asked to explain aloud or not (e.g., Berardi-Coletta, Buyer, Dominowski, & Rellinger, 1995).

1.3. How comparison and explanation affect learning

Given that both contrasting cases and self-explanation are beneficial for learning, it would be valuable to know whether comparison and explanation have distinct effects. Put another way, do contrasting cases instruction and self-explanation affect different aspects of the learning process? The primary aim of this study is to examine whether comparison and explanation have independent effects on student learning.

To address this aim, we needed to separate comparison and explanation in our research design. At a theoretical level, we distinguish noticing similarities and differences between analogous problems, which we call *comparison*, and making sense of problems, which we call *explanation*. Of course, in everyday circumstances, when learners compare problems, they often attempt to make sense of the similarities and differences that they notice; thus, comparison and explanation often co-occur. Furthermore, instruction that focuses on comparison often includes prompts to explain, (e.g., “How do you know that both ways to solve the problem are correct?”; Rittle-Johnson et al., 2012), and therefore promote explanations that include comparison. Some studies of explanation provide prompts that direct learners to compare exemplars (e.g., “Explain why robots 1, 2, 3, and 4 might be Outdoor robots …”; Williams & Lombrozo, 2013) or to compare an exemplar to a category (e.g., Hefter et al., 2014; Williams & Lombrozo, 2010). Thus, in practice, explanation and comparison often overlap. In our view, however, it should be possible to distinguish these processes empirically. That is, participants could engage in *noticing* similarities and differences between examples without attempting to *explain* those similarities and differences.

It could be argued that comparison is a type of self-explanation, because some sense-making may be involved in noticing similarities and differences. However, we believe that learners *can* notice similarities or differences without making sense of those similarities and differences (i.e., pure perceptual comparison) or without explicitly considering the implications of those similarities and differences. Therefore, we do not consider comparison a type of self-explanation. Whether learners *do* engage in explanation when they compare is an open empirical question that we sought to address in this study.

In our research design, we manipulated comparison and explanation separately. To elicit comparison, we asked participants simply to *report* whether there were similarities and/or differences between the presented examples; we did not ask them to identify, describe, or otherwise account for those similarities or differences. To elicit self-explanation, we asked participants to *explain* and *make sense* of the problems. Of course, participants who were not asked to compare or self-explain may have spontaneously engaged in these processes; to address this possibility, we examined participants' verbal protocols for evidence of spontaneous comparison and explanation.

1.3.1. Comparison

Contrasting cases instruction may influence learning by allowing students to differentiate important problem features. According to theories of perceptual learning, opportunities to compare problems can highlight shared features (Gagne & Gibson, 1947; Gick & Paterson, 1992). Comparing problems can also help learners differentiate critical shared features from otherwise alluring, but non-critical features (e.g., Schwartz & Bransford, 1998). Although superficial differences are often noticed first, a closer comparison often reveals more relevant, deeper similarities (Cummins, 1992; Marton, 2006).

By comparing examples with similar underlying structures, despite superficial variations, learners may extract a common relational structure (Schwartz et al., 2011). Students may recognize problems with similar relational structures as members of a category of problems, implying that they have a similar conceptual basis (e.g., Hattikudur & Alibali, 2010) and can be solved with similar procedures (e.g., Schwartz et al., 2011). This knowledge supports transfer of knowledge to problems with different surface features but shared structure.

1.3.2. Explanation

Prompts to self-explain may affect learning in at least two ways (Chi, 2000). First, self-explanations may support students' *filling gaps* in the learning materials by generating inferences. Worked examples often contain information about how to solve a problem, but lack information about why the procedure was suited for that problem or what domain principles guided its application. Good self-explainers fill in these gaps by making inferences. Second, self-explanations may help learners *generate and revise mental models*. Learners often have some prior knowledge of the domain, and that knowledge may be correct or incorrect. In self-explaining, learners may recognize their lack of understanding or detect inconsistencies between their reasoning and given information, prompting them to repair or abandon misconceptions or to generate new knowledge. Along similar lines, Lombrozo (2012) argues that everyday explanations help learners apply their prior knowledge and beliefs in making sense of new experiences.

Some researchers have suggested that the specific mechanisms at play depend on the structure of the domain and the learner's prior knowledge. According to the *instructional fit* hypothesis (Nokes, Hausmann, VanLehn, & Gershman, 2011), explanation prompts are more successful when the aim of the prompt fits the structure of the domain and the learner's knowledge state. Nokes et al. (2011) compared learning from prompts that encouraged either gap-filling or mental model revision. In the domain of electrodynamics, for which students were unlikely to have prior misconceptions, gap-filling prompts led to better performance, compared to prompts that encouraged mental model revision. In contrast, in learning about the circulatory system—a domain in which students often hold misconceptions, but for which a coherent mental model supports causal reasoning—many students' explanations reflected mental model revision (Chi, 2000). Thus, self-explanation can support learning in different ways, depending on the structure of the domain and learners' existing knowledge.

1.3.3. Unique or interactive effects?

Comparison and explanation both support learning from worked examples. However, it remains unclear whether observed learning gains in studies of contrasting cases instruction are due primarily to explanation, primarily to comparison, or to their combination. If comparison and self-explanation operate via different mechanisms (e.g., identifying structural features vs. gap-filling), combining them should produce additive effects. Alternatively, some mechanisms (e.g., highlighting common relational structure and revising mental models) could interact. For example, contrasting cases could support extraction of a common relational structure, and attention to this structure could lead to richer self-explanations, in turn supporting better mental model revision. Indeed, Gadgil, Nokes-Malach, and Chi (2012) found that learners who explained comparisons between their own flawed mental models and expert mental models showed more mental model revision than those who explained only an expert model. In this study, we attempt to disentangle the effects of explanation and comparison by manipulating explanation and comparison independently, and investigating the mechanisms by which instruction involving these activities promotes learning.

1.4. Current study

This study addresses three questions. First, what are the unique and combined effects of self-explanation prompts and contrasting cases instruction on students' learning? We hypothesized that both self-explanation and contrasting cases would promote learning. Second, how do self-explanation prompts and contrasting cases instruction, separately and in combination, influence learning processes? Based on past work, we hypothesized that self-explanation would foster gap-filling or mental model revision, and contrasting cases would foster identifying key structural features of the problems. Third, are specific learning processes responsible for the effects of self-explanation prompts and contrasting cases instruction on learning? We explored whether specific learning processes mediated the effects of self-explanation and contrasting cases on learning.

1.4.1. Target domain: fraction division

We address these questions in students learning fraction division. Fraction understanding is an integral component of arithmetic knowledge, as reflected in the Common Core State Standards (NGA Center & CCSSO, 2010). Moreover, children's early fraction knowledge uniquely predicts high school mathematics achievement, even after controlling for prior academic achievement and many other factors (Siegler et al., 2012).

However, many adults and children have poor knowledge about fraction division. Some learners lack *procedural knowledge* of fraction division, defined as knowledge of procedures for solving problems (e.g., Siegler, Thompson, & Schneider, 2011), and many learners also lack *conceptual knowledge*, defined as understanding of foundational concepts and of the basis for solution procedures (Crooks & Alibali, 2014). Relatively few studies have directly measured conceptual understanding of fraction division in children (though see Hattikudur, 2012; Sidney & Alibali, 2014); however, adult data suggest that even prospective elementary teachers lack conceptual understanding of fraction division (e.g., Lo & Leu, 2012). For example, Ma (1999) asked teachers to generate a story to represent a fraction division expression; only one of 21 American teachers succeeded.

1.4.2. Overview

In the current study, participants studied worked examples of whole number division and fraction division problems. We utilized both whole number and fraction division because past work has shown that activating whole number division concepts can support understanding of fraction division (Sidney & Alibali, 2014).

Students studied the worked examples either with or without prompts to self-explain. By including worked examples of both fraction and whole number division, we sought to support the integration of fraction division with prior division knowledge, particularly for participants asked to self-explain.

Students studied the examples either individually or in sets. Students who viewed the problems individually were presented the problems sequentially, each on a separate page, with no instructions to compare, so as to minimize comparison between problems. Students who viewed the problems in sets were presented each set of problems together on a single page, and each set included both whole number division and fraction division. This contrasting cases instruction was designed to provide students opportunities to discover the underlying structure that was similar across cases (i.e., the quotative relationship among the dividend, divisor, and quotient).

We hypothesized that both comparison and self-explanation would lead to improved conceptual understanding of fraction division. We also examined whether comparison and self-explanation affected later learning of a fraction division procedure.

2. Method

2.1. Participants

Participants were 81 undergraduates (51 women, 30 men) from an introductory psychology course at a public university in the Midwestern USA. Per self-report, the sample was 71% White, 20% Asian, 5% Hispanic, 3% Black, and 1% Native American.

2.2. Design

The study utilized a 2 (self-explanation or no self-explanation) x 2 (contrasting cases or sequential cases) between-subjects design. Therefore, participants were randomly assigned to one of four conditions: *control* (no self-explanation prompt, sequential cases), *self-explanation only* (self-explanation prompt, sequential cases), *contrasting cases only* (no self-explanation prompt, contrasting cases), and *combined* (self-explanation prompt, contrasting cases).

2.3. Procedure

Each participant completed one individual session lasting 30–45 min. First, participants completed a pretest measure of their fraction division knowledge. Then, participants studied a series of worked examples, during which the experimental manipulation took place. Unlike other worked example studies, we did not first introduce the domain, because undergraduates are familiar with fraction division. While studying, all participants were asked to talk aloud (i.e., provide concurrent verbalizations). Importantly, the talk-aloud instructions did not include instructions to explain, following Ericsson and Simon's (1993) recommendations for eliciting concurrent verbalizations. Past research has shown that when people are instructed to *explain* their thinking, rather than simply *verbalize* their thought processes, they engage in additional cognitive processes that alter their task performance (Ericsson & Simon, 1993).

All participants studied the same four worked examples. Participants in the *control* condition were asked to study each problem while copying the equation and picture, in order to keep their attention focused on the problems during the study period. Participants in the *self-explanation only* condition were asked to "explain the problem to yourself, out loud, and try to make sense of what you see". Participants in the *contrasting cases only* condition received the problems in sets (described below). Participants were asked to study each example and then report whether there were similarities and differences between the problems in each set. Participants in the *combined* condition followed the same procedure as those in the *contrasting cases only* condition, but were also given the self-explanation prompt. In all conditions, the talk-aloud prompt was repeated as each new page of problems was presented, and the experimenter reminded participants to study the problems for the entire allotted time, which was 6 min in all conditions.

After studying the problems, participants completed a filler activity for 1 min. Next, all participants studied a procedure for fraction division, described below. Finally, participants completed a fraction division posttest.

2.4. Lesson materials

2.4.1. Worked example problems

The worked examples highlighted the quotative structure of fraction division in two ways. First, each problem was represented using a diagram that showed the dividend as a number of whole objects, the divisor as the grouping unit (the size of each group), and the quotient as the number of groups (Fig. 1). Second, two of the problems involved whole number division and two involved fraction division. The whole number division problems could cue the division grouping structure shared across all problems.

In the sequential cases conditions (*control* and *self-explanation only*), each problem was printed on a separate page, to inhibit comparison. In the contrasting cases conditions (*contrasting cases only* and *combined*), participants studied the problems in the following sets: (1) the first whole number division problem and the first fraction division problem, (2) the second whole number division problem and the second fraction division problem, and (3) all four previously presented problems. Each set was presented on one page. At the bottom of each page, participants were asked to circle "yes" or "no" to indicate whether there were similarities between the problems in that set and whether there were differences between the problems in that set (Fig. 1). Importantly, participants were *not* instructed to identify or describe the similarities or differences. With this manipulation, we sought to encourage comparison without eliciting explanation.

2.4.2. Fraction division procedure

Following the worked examples, participants studied a novel fraction division procedure (Fig. 2). The procedure manipulates a fraction division expression in a way that results in multiplying by the inverse of the divisor, but that includes intermediary steps to highlight that the inverse corresponds the number of times the divisor goes into 1. For example, in 6 ÷ ½, multiplying by the inverse corresponds to finding the number of times ½ goes into 1 and then multiplying this result by the dividend. Thus, the procedure highlights the grouping concept (i.e., how many groups of ½ are in 1, and subsequently, 6), although it did not make this connection explicit in any way.

We hypothesized that activating students' knowledge of whole number division might prepare them to learn this version of the invert-and-multiply procedure. Thinking about whole number

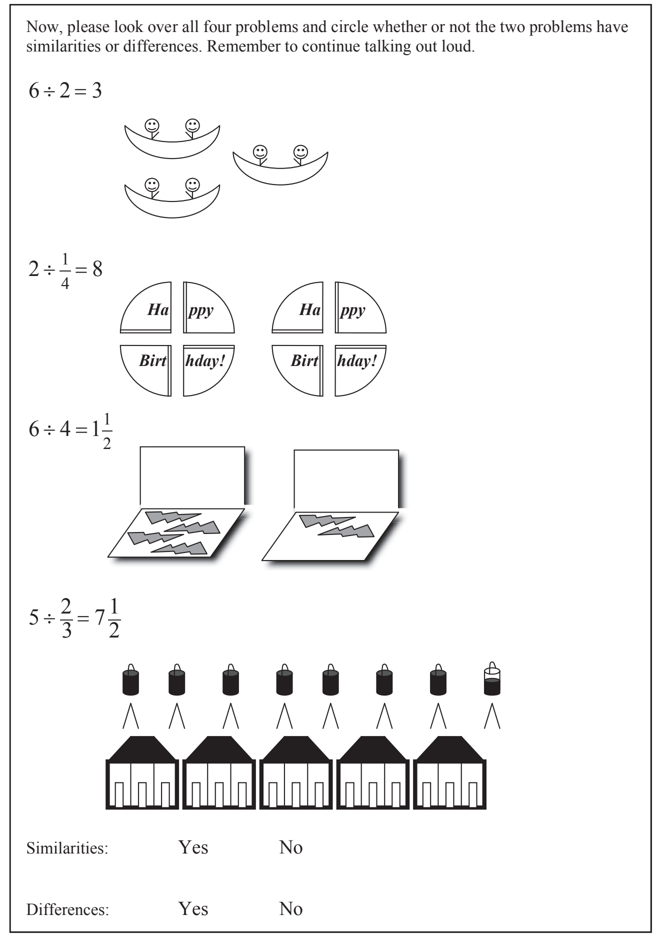

Fig. 1. The four study problems, as shown on the last worksheet given to participants in the *contrasting cases only* and *combined* conditions. Participants in the *control* and *self-explanation only* conditions received the same problems in the same order, but individually, on separate sheets.

Please study the following procedure for solving this problem.

$$a \div \frac{b}{c} =$$

Step 1: $a \times 1 \div \frac{b}{c} =$

Step 2: $a \times \frac{c}{b} \times \frac{b}{c} \div \frac{b}{c} =$

Step 3: $a \times \frac{c}{b} \times 1 =$

Step 4: $a \times \frac{c}{b} =$

Fig. 2. The invert-and-multiply procedure that participants studied during the lesson.

division could activate the idea of "how many times" the divisor goes into the dividend. Students could connect this idea to "how many times" the fractional divisor (in our example, ½) goes into 1, and they might then realize that this division occurs for *each* of the units in the dividend (e.g., in 6 ÷ ½, it occurs 6 times, so one multiplies by 6). As such, we explored whether connecting knowledge of whole number division and fraction division (via self-explanation and/or contrasting cases) would support students' learning of this procedure.

2.5. Knowledge measures

2.5.1. Pretest and posttest

At pretest and posttest, we assessed both conceptual and procedural knowledge of fraction division. The pretest included three items assessing procedural knowledge (equation solving items) and one item assessing conceptual knowledge (story generation item). The posttest included seven items assessing procedural knowledge (equation solving items and procedural recall items) and two items assessing conceptual knowledge (story generation item and picture generation items).

2.5.2. Procedural knowledge

Participants demonstrated procedural knowledge by solving equations and (at posttest) recalling the taught procedure. The pretest included three equations with unit fractions ($5 \div 1/4 = ?, 6 \div 1/3 = ?, 1/3 \div 1/6 = ?$). The posttest included two equations with non-unit fraction divisors ($3 \div 3/5 = ?, 7 \div 3/4 = ?$), two with mixed fraction dividends ($21/3 \div 41/6 = ?, 2/3 \div 4 = ?$), and one missing operand problem ($6 \div ? = 24$). The posttest also included two procedural recall items. On these items, participants were asked to reproduce each step of the procedure they studied during the lesson, first with letters representing unknown quantities (($x \div y/z=$, similar to the lesson itself) and then with numbers ($3 \div 1/6=$).

2.5.3. Conceptual knowledge

Conceptual knowledge was assessed using story and picture generation items. Each test included one story generation item instructing participants to write a story to correspond to a fraction division expression (e.g., 6 ÷ 1/3) (similar to items used by Ma, 1999). Each story received one point for representing a division relationship between the dividend and the divisor, and one point for representing the correct quotient (possible total of two points). To illustrate, for (6 ÷ 1/3), a correct story (two points) would represent six as a given quantity (e.g., number of pizzas), represent one-third as the grouping unit (e.g., one serving is one-third of a pizza), and ask how many grouping units there are in the given quantity (e.g., How many servings of pizza?).

The posttest also included one picture generation item. Participants were asked to draw a picture to correspond to a fraction division expression (4 ÷ ½). Drawings were coded as correct if they represented four objects each being split into two equal parts (quotative division) or four objects comprising one half of a larger group (partitive division). Correct drawings received two points, and drawings representing the correct quotient but incorrect division relationship received one point.

2.6. Coding talk-aloud protocols

Participants in all conditions provided talk-aloud protocols while studying the problems. We coded these protocols for three processes that have been implicated in the self-explanation literature: knowledge generation, knowledge access, and monitoring. We also coded for evidence of spontaneous comparison. Three videos were lost due to equipment failure. Two independent coders coded 25% of the records to establish reliability: agreement was 90% for knowledge generation, 83% for knowledge access, and 90% for monitoring.

2.6.1. Knowledge generation

To evaluate knowledge generation, we coded talk-aloud protocols for whether each participant correctly identified the roles of the dividend, divisor, and quotient. In addition, for each problem, we coded three additional sub-types of knowledge generation: explicitly applying a correct grouping or unit structure to the problem (e.g., "*each group has two people*"), interpreting an action in the diagram (e.g., "*you need to put two people into each boat*"), and posing a problem-solving question about the diagram that corresponded to the equation structure (e.g., *how many boats will you need for six people?*).

Each form of knowledge generation (i.e., identifying dividend, identifying divisor, identifying quotient, applying a grouping/unit structure, interpreting action, and posing an appropriate problem-solving question) was coded as occurring or not on each worked example problem, resulting in a possible score of 6 on each problem. An overall knowledge generation score was calculated for each participant by summing these scores across all four problems ($M = 9.14, SD = 4.44$, range 0–18).

For analyses of mental model revision, we also created a *structure score* for each of the fraction division problems by summing scores for identifying dividend, identifying divisor, and identifying quotient for each problem.

2.6.2. Knowledge access

We coded talk-aloud protocols for evidence of accessing related prior knowledge. Typically, this occurred when a participant stated a known mathematical fact (e.g., *two times three is six*) or applied a previously-learned procedure. Knowledge access was coded as occurring or not on each problem.

2.6.3. Monitoring

Monitoring statements were coded when participants reflected on their understanding, either positively (e.g., *Oh, I get it now*), negatively (e.g., *I'm not sure what this means*), or via self-questioning (e.g., *did that make sense?*). Monitoring was coded as present or absent for each problem.

2.6.4. Comparison

Finally, we coded talk-aloud protocols for evidence of spontaneous comparison. We coded for any explicit reference to a previous problem. This occurred only once; therefore, this code was not included in the analyses reported here.

3. Results

3.1. Effects of self-explanation and comparison on knowledge gains

We hypothesized that self-explanation and contrasting cases would lead to deeper conceptual understanding and better learning of a novel procedure for fraction division. To analyze the data, we used 2 x 2 between-subjects ANCOVAs, controlling for total pretest scores, for each posttest measure. Pretest scores did not differ significantly across conditions (Table 1).

3.1.1. Procedural knowledge

To examine procedural knowledge gains, scores on the posttest procedural items were summed for each participant to create a procedural posttest score (range 1–7, $M = 4.73$, $SD = 1.72$). No effects were significant in the ANCOVA analysis; there was no main

Table 1
Mean scores by experimental condition.

Condition	Pretest	Procedural posttest	Conceptual posttest	Knowledge generation	Knowledge Access	Monitoring
Control	2.90 (1.89)	4.86 (.36)	2.27 (.28)	9.01 (1.03)	.68 (.23)	.68 (.23)
CC Only	3.52 (1.54)	4.68 (.36)	2.31 (.28)	3.36 (1.12)	.81 (.23)	.54 (.23)
SE Only	1.95 (1.75)	4.66 (.36)	2.88 (.28)	10.41 (.89)	1.01 (.22)	1.17 (.22)
Combined	2.89 (1.85)	4.70 (.37)	3.13 (.29)	10.51 (1.04)	.90 (.23)	1.05 (.23)

Note. Outcome means are calculated controlling for variance due to pretest scores. Standard errors are in parentheses.

effect of self-explanation, $F(1,76) = .06$, $p = .81$, no main effect of contrasting cases, $F(1,76) = .04$, $p = .85$, no interaction, $F(1,76) = .10$, $p = .75$ (Table 1). Thus, procedural learning did not vary as a function of condition.

3.1.2. Conceptual knowledge

To examine conceptual knowledge gains, scores on the posttest conceptual items were summed for each participant to create a conceptual posttest score (range 0–4, $M = 2.64$, $SD = 1.40$). There was a significant main effect of self-explanation on conceptual posttest scores, $F(1,76) = 6.30$, $p = .01$, $\eta_p^2 = .08$. Participants who were prompted to self-explain demonstrated greater conceptual knowledge at posttest ($M_{SE} = 3.00$, $SE_{SE} = .20$) than those not prompted to self-explain ($M_{NoSE} = 2.29$, $SE_{NoSE} = .20$; Fig. 3). There was no main effect of contrasting cases, $F(1,76) = .26$, $p = .61$, and no interaction, $F(1,76) = .13$, $p = .72$. Thus, self-explanation prompts fostered conceptual learning, but contrasting cases instruction did not.

3.2. Did explanation affect learners' comparisons?

The foregoing analyses indicate that studying contrasting cases did not affect understanding of fraction division independent of self-explanation, implying that explanation may play a critical role in the comparison process. We next examined effects of self-explanation condition on the *quality* of comparisons. On each comparison worksheet, participants were asked to indicate whether or not they noticed similarities between the problems (yes or no) and differences between the problems (yes or no). Participants who engage in deeper processing of the examples should be more likely to notice *both* similarities and differences (Gentner & Gunn, 2001). Since participants in the sequential cases conditions were not asked to report similarities and differences at all, those participants were excluded from this analysis.

To examine whether self-explanation prompts affected comparison processes, we examined the effects of self-explanation prompts (between subjects) and question type (similarities or differences; within subjects) on the number of "yes" responses to the noticing questions, controlling for pretest scores, using 2 × 2 mixed ANCOVA. As seen in Fig. 4, participants who were prompted to self-explain noticed more similarities and differences than those not prompted to self-explain, $F(1,37) = 12.96$, $p < .001$, $\eta_p^2 = .26$. There was no main effect of question type (similarities, differences), $F(1,37) = .04$, $p = .85$, nor was there an interaction of self-explanation condition and question type, $F(1,37) = .01$, $p = .92$. Pretest scores also did not predict variation in noticing similarities and differences, $F(1,37) = .09$, $p = .77$.

3.3. Effects of explanation and comparison on learning mechanisms

Given the effects of self-explanation prompts on both conceptual learning and on the comparison process itself, we sought to identify the mechanisms by which these prompts supported understanding of the problems. We also examined whether the contrasting cases manipulation affected the explanation process. Thus, we estimated the effects of the self-explanation prompts and contrasting cases manipulations on the three dimensions of speech coded from the talk-aloud protocols (knowledge generation, knowledge access, monitoring) in a series of models regressing scores for each dimension on self-explanation (coded .5 for self-explanation and −.5 for no self-explanation), contrasting cases (coded .5 for contrasting cases and −.5 for sequential cases), and their interaction, controlling for total pretest scores, mean-centered. Correlations among knowledge generation, knowledge access, and monitoring scores are presented in Table 2.

We also sought to assess whether participants engaged in mental model revision, by examining change in understanding of problem structure from the first to the second fraction division problem during the study period.

3.3.1. Knowledge generation

Both self-explanation, $b = 4.27$, $t(73) = 5.29$, $p < .01$, $\eta_p^2 = .28$, and contrasting cases, $b = -2.77$, $t(73) = 3.41$, $p < .01$, $\eta_p^2 = .14$, affected participants' knowledge generation scores. Furthermore, there was a significant interaction, $b = 5.75$, $t(73) = 1.59$, $p < .01$,

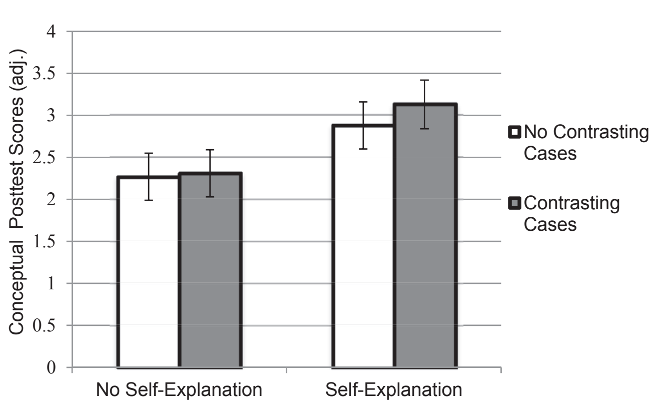

Fig. 3. Conceptual posttest scores, controlling for pretest scores, by condition. Error bars reflect standard errors of point estimates.

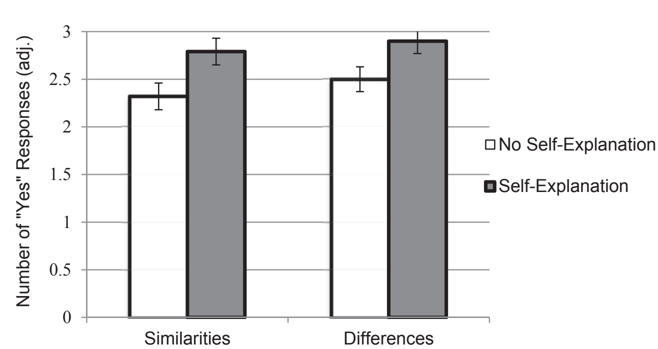

Fig. 4. Number of "yes" responses to questions about noticing similarities and differences, controlling for pretest, for participants in the contrasting cases conditions. Error bars reflect standard errors of point estimates.

Table 2
Correlations among think-aloud measures and conceptual posttest scores.

	Knowledge generation	Knowledge Access	Monitoring
Knowledge Access	−.06 (.61)	–	–
Monitoring	.23* (.04)	.23* (.04)	–
Conceptual Posttest	.03 (.81)	.14 (.23)	.05 (.75)

Note. All $ns = 78$; p-values in parentheses.
*Estimates are significant at $\alpha = .05$.

$\eta_p^2 = .15$(Fig. 5). Among participants in the self-explanation conditions, knowledge generation scores did not differ for those who explained with contrasting cases and those who explained with sequential cases, $t(73) = .10$, $p = .93$. However, among participants who were *not* asked to self-explain, those who received contrasting cases had *lower* knowledge generation scores than those who received sequential cases, $t(73) = -4.94$, $p < .01$ (Table 1).

3.3.2. Knowledge access

In contrast, neither self-explanation, $b = .21$, $t(73) = .91$, $p = .37$, nor contrasting cases, $b = .01$, $t(73) = .03$, $p = .98$, affected the number of problems on which participants produced statements indicating that they accessed prior knowledge, nor was there a significant interaction, $b = -.25$, $t(73) = -.55$,$p = .59$(Table 1).

3.3.3. Monitoring

Participants prompted to self-explain expressed monitoring statements on more problems ($M = 1.17$, $SE = .22$) than those not prompted to self-explain ($M = .66$, $SE = .26$), regardless of contrasting cases condition, and controlling for pretest scores, $b = .50$, $t(73) = 2.21$, $p = .03$. Contrasting cases did not affect monitoring, $b = -.13$, $t(73) = -.58$, $p = .56$, and there was no significant interaction, $b = .03$, $t(73) = .06$, $p = .95$(Table 1).

3.3.4. Mental model revision

We coded mental model revision by analyzing change in participants' identification of key structural features from the first to the second fraction division problem. However, because the second problem was much more difficult than the first problem, participants were actually *worse* at identifying structure on the second problem than the first ($M = 1.65$ vs. $M = 2.22$ of 3 elements identified). Therefore, we classified participants as being above or below the overall median in structure score for each problem, and looked at change only among those who were below the median on the first problem.

Participants who were prompted to self-explain were substantially less likely to be below the median on the first problem (6 of 40) than those not prompted to self-explain (26 of 38; $p < .001$, Fisher's Exact), suggesting that the self-explanation prompt fostered participants' accurately encoding problem structure (as expected, given the main effect of self-explanation on knowledge generation). This made it challenging to discern whether participants engaged in mental model revision, because most participants in the self-explanation conditions displayed accurate mental models already on the first problem. Among participants whose structure scores were below the median on the first problem, 50% (3/6) of those prompted to explain were above the median on the second problem, thus demonstrating mental model revision during the study period. In contrast, only 27% (7/26) of those not prompted to explain were above the median on the second problem. These data are compatible with the idea that self-explanation encourages mental model revision. However, because so few participants in the self-explanation conditions had "room" to display mental model revision, they should be interpreted with caution.

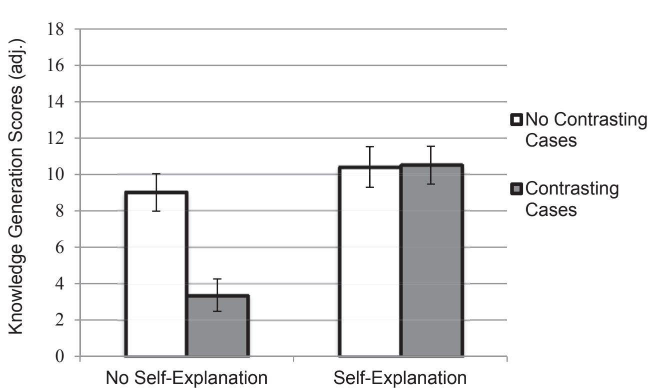

Fig. 5. Mean knowledge generation scores, controlling for pretest scores, by condition. Error bars reflect standard errors of point estimates.

3.4. Effects of learning mechanisms on conceptual knowledge

Participants who were prompted to self-explain did indeed engage in processes hypothesized to support the self-explanation effect, specifically, knowledge generation and monitoring. Therefore, we tested whether knowledge generation or monitoring mediated the effects of self-explanation on conceptual learning, using the *mediation* package (Tingley, Yamamoto, Keele, & Imai, 2013) in R (R Core Team, 2014). Preacher and Hayes (2004) argue that demonstrating a mediated effect requires two preconditions: that there is a statistical effect to be mediated and that the indirect effect via the mediator is significant. Given the significant effect of self-explanation condition on conceptual learning, we then estimated the average causal mediated effects (ACME; the indirect effects) using a non-parametric bootstrapping procedure, as recommended by Preacher and Hayes, for each potential mediator, separately. In the *mediation* package, the ACME and its bootstrapped 95% confidence interval are estimated by resampling the data 1000 times and computing the indirect effect in each sample. The 95% CIs of the ACMEs included zero in both models (knowledge generation as mediator, $ACME = -.14[-.49, .14]$; monitoring as mediator, $ACME = .01$ $[-.16, .18]$). Thus, contrary to expectations (and to much of the literature on self-explanations), there was no evidence that either knowledge generation or monitoring mediated the effect of self-explanation on conceptual learning.

We also tested whether mental model revision mediated the effect of self-explanation on conceptual learning among participants who had low structure scores on the first fraction division problem, and who therefore had room to change ($n = 32$). Participants were classified as either demonstrating mental model revision between the first and second fraction division problems or not. First, we assessed the effect of mental model revision on conceptual learning, after controlling for self-explanation; this path did not differ significantly from zero, $b = .23$, $p = .66$. Next, we estimated the ACME of self-explanation on conceptual learning through mental model revision. The 95% CI included 0, $ACME = .05[-.23, .47]$. Thus, there was no evidence that the effect of self-explanation on conceptual learning was mediated by mental model revision.

4. Discussion

4.1. Direct effects of comparison and explanation on learning

This study evaluated the unique and combined effects of explanation and comparison on students' learning of fraction division. Comparison is a widely used tool in mathematics instruction; we examined whether its effectiveness depends on its incorporating explanation. Contrasting cases instruction, in the absence of a self-explanation prompt, did not affect learning of fraction division. This finding suggests that the reported benefits of contrasting cases on mathematics learning may be largely due to the inclusion of explanation prompts in most studies. One possible reason is that explanation imposes a *subsumptive* constraint, leading participants to consider how an example fits into a more general category, defined by structural features (Williams & Lombrozo, 2010). From this perspective, our findings suggest that explanation may *potentiate* the power of comparison for highlighting shared structure.

More surprisingly, however, we found that encouraging comparison did not promote conceptual learning over and above prompts to self-explain. This finding contrasts with Gadgil and colleagues' (2012) finding that facilitated comparisons improved mental model revision over and above explanation alone. However, Gadgil et al. also supported learners' explanations of the comparisons, whereas we examined students' self-generated explanations.

One possible explanation for the non-effect of contrasting cases that we observe in this study is that learners found it challenging to structurally align whole number and fraction division problems without support. The relevant comparisons may have been too difficult. If learners are unsuccessful in aligning the items, it is not surprising that comparison is ineffective. Consistent with this view, participants produced very few utterances indicating comparison, even when encouraged to self-explain.

A second possibility lies in participants' prior knowledge. Though adults tend to have weak knowledge of both whole number division (e.g., Dixon, Deets, & Bangert, 2001) and fraction division (e.g., Ma, 1999), these problem domains are not novel. Indeed, arithmetic as a domain is highly practiced. If contrasting cases helps learners to differentiate structural and surface attributes of worked examples, contrasting cases may have been ineffective because students' existing representations were already “good enough” to support this distinction. Contrasting cases instruction may be particularly useful in introducing learners to novel domains, rather than supporting successful revision of prior knowledge in familiar domains.

A third possibility has to do with the difficulty of the novel procedure we taught. This procedure was complex, and it was challenging for participants to discern its conceptual basis. This difficulty could also account for the non-effect of contrasting cases on procedural learning.

Note that all of these possible explanations have to do with the nature of fraction division. These lines of argument suggest that the non-effect of contrasting cases instruction may not generalize to other mathematical domains, particularly ones in which problems are easier to align or where relevant procedures are less complex. Put another way, fraction division may have features that constrain participants' abilities to benefit from comparison.

However, along similar lines, it is possible that beneficial effects of self-explanation that we observed may also be due, at least in part, to the nature of the domain. Self-explanation is thought to directly influence mental-model revision (e.g., Chi, 2000) in domains in which learners have misconceptions, and fraction division is one such domain.

4.2. Effects of comparison and explanation on learning processes

Our secondary goal was to better understand the ways in which comparison and explanation affect learning processes. Since comparison and explanation are thought to affect learning via different mechanisms, we hypothesized that contrasting cases instruction and prompts to self-explain might have unique effects that could interact to enhance learning, over and above the additive effects of each approach individually. Although we did not find such interactive effects, we did find that self-explanation prompts affected students' comparison processes and that contrasting cases instruction affected the quality of students' explanations.

4.2.1. Prompting self-explanations affects comparison

Prompts to self-explain changed the nature of students' comparisons. Learners who were asked to self-explain noticed more similarities and differences across problems, suggesting more thorough alignment (e.g., Gentner & Gunn, 2001). Both similarities and differences are highlighted when learners make a *relational* match across shared conceptual structure, rather than an *attribute*-based match across surface features (Medin, Goldstone, & Gentner, 1990). In line with this view, we suggest that noticing more similarities and differences in response to the contrasting cases manipulation indicates that participants in the combined self-explanation and contrasting cases condition attended more to the relational, quotative structure across problems. Surprisingly, however, we did not find benefits of contrasting cases instruction, on its own, on conceptual learning.

4.2.2. Providing opportunities for comparison affects explanations

Providing students with contrasting cases moderated the effect of the general self-explanation prompt on the quality of students' talk during study. Unexpectedly, however, learners who were given contrasting cases and asked to report similarities and differences in the absence of the self-explanation prompt engaged in *less* gap-filling (as captured by our knowledge generation measure) than learners in any other instructional condition. These students presumably noticed similarities and differences, but did not engage in making sense of those similarities and differences. Thus, in the absence of instructions to *explain* similarities and differences, simply *noticing* similarities and difference did not help students to deeply comprehend worked examples.

Taken together, our results suggest that explanation is a critical element of learning from contrasting cases instruction. Indeed, most demonstrations of the effectiveness of contrasting cases instruction in the literature involve both comparison *and* explanation. We suggest that it is not the juxtaposition of examples or the encouragement to notice similarities and differences that promotes learning. Instead, contrasting cases instruction supports learning when it incorporates either direct or indirect encouragement to explain.

4.3. Effects of Proposed mechanisms on learning

Our third goal was to understand how the specific learning mechanisms involved in comparison and explanation might mediate any effects on learning. Learners who were asked to self-explain were more likely to generate new knowledge, and specifically, to engage in helpful gap-filling. In addition, self-explanation promoted monitoring, consistent with past work (e.g., Wong, Lawson, & Keeves, 2002) as well as with Chi's (2000) mental model revision hypothesis. Self-explanation prompts may encourage students to actively monitor their understanding of worked examples. In our study, however, the frequency of knowledge generation and monitoring statements did not predict conceptual learning.

Like us, Wong et al. (2002) found that monitoring did not mediate the effect of self-explanation on learning. However, in contrast to our findings, Wong et al. found that students who were trained to self-explain while studying geometry problems generated more knowledge and accessed their own prior knowledge at higher rates than untrained students, and these processes mediated the effect of self-explanation on learning. Along similar lines, Berthold, Eysink, and Renkl (2009) found that the content of learners' explanations mediated the effects of self-explanation on conceptual learning from worked examples of probability problems. In contrast, we found no relationship between knowledge generation and conceptual knowledge.

What could account for our discrepant findings? One possibility is that differences in the target domains—geometry, probability, and fraction division—may be responsible. However, another striking difference across studies is in the nature of the prompts. We used a “bare” self-explanation prompt, to be consistent with the explanation prompts typically embedded in contrasting cases instruction. Wong et al. (2002) and Berthold et al. (2009) provided additional instructional support for learners' self-explanations through training and computer-based support, respectively, to enhance explanation content.

Furthermore, as discussed above, learners entered our study with substantial prior knowledge, and presumably with misconceptions about fraction division. Thus, consistent with the *instructional fit hypothesis* (Nokes et al., 2011), fraction division may be a domain in which self-explanation affects learning via mental model revision, and not gap-filling. Indeed, we measured learners' gains in conceptual knowledge by their ability to produce representations (i.e., stories and pictures) that were consistent with a conceptually correct model of division. Therefore, even though self-explanation increased gap-filling (i.e., knowledge generation) in our worked examples, this gap-filling may have been less critical to conceptual learning than mental-model revision. We sought to identify mental model revision across the study problems, but we were unable to do so definitively, because most participants who were asked to self-explain correctly identified the structure of fraction division on the first problem. For these students, mental model revision may have occurred in the process of explaining the first fraction division problem.

In brief, our findings leave open the question of the specific mechanism that drives the effect of explanation on conceptual learning of fraction division. In future studies, a more targeted measure of mental model revision should be used, so that firmer conclusions can be drawn about whether self-explanation affects conceptual learning via mental model revision. Future studies should also examine other hypothesized mechanisms by which self-explanation could foster learning, such as by helping learners to discover general patterns (Williams & Lombrozo, 2013).

5. Conclusions

When studying whole number and fraction division problems, learners gained more conceptual knowledge when asked to explain, regardless of whether or not they were encouraged to compare across problems. We expected that juxtaposing whole number and fraction division and encouraging participants to make comparisons would help participants understand fraction division as a type of division, but we found no evidence that comparison on its own (i.e., in the absence of an explanation prompt) was beneficial.

We suggest that comparison may be most effective for novel domains. In the domain this study targeted, contrasting cases instruction was not especially useful, presumably because students had substantial prior knowledge. Nevertheless, we believe that the observed interplay between contrasting cases and self-explanation sheds light on how explanation and comparison might combine to yield larger effects on learning in more novel domains. Our findings underscore that explanation is a critical aspect of contrasting cases instruction, and that explanation may underlie the documented benefits of instruction that involves comparison. Put simply, explanation potentiates comparison.

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