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Mathematics in the home: Homework practices and mother–child interactions doing mathematics

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Abstract

Parents are a largely untapped resource for improving the mathematics performance of American children, which lags behind the performance of children from other nations. The purpose of the research reported here was to assess homework practices in the home, and to examine interactions between mothers and their 5th grade children as they worked challenging mathematics problems (pre-algebra equivalence problems). Results indicated that children spent on average 23 min per day on mathematics homework, with an average of 8 min of help from parents. Videotapes of mother–child interactions indicated that mothers varied considerably in the quality of the mathematics content that they conveyed while teaching, and in the quality of their scaffolding of the material for the child. As expected, mothers who themselves had more mathematics preparation performed better in conveying mathematical content and in scaffolding. Mothers with more mathematics self-confidence also performed better. The results suggest that children face inequities in the parental resources available to them for math learning; these inequities might be remedied by school–family partnership programs.

Keywords: Mother–child interactions; Mathematics; Homework; School–family partnership

Experts continue to decry the poor performance of American children on standardized mathematics tests (National Education Goals Panel, 1997; Stevenson, Chen, & Lee, 1993). Hundreds of studies have investigated mathematics learning in the formal setting of the school, including analyses of teacher–student interactions (Brophy & Good, 1974; Fennema & Peterson, 1986; Koehler, 1990) and best methods for teachers to use in instruction (e.g., Carpenter & Fennema, 1992; Henningsen & Stein, 1997). In contrast, informal mathematics learning in the home has received little research attention. Parents' contributions to their children's mathematics learning are a largely untapped resource for improving the mathematics performance of American children. Before that resource can be maximized, however, basic research is needed on the nature of these parent–child interactions. The primary purpose of the research reported here was to assess mathematics homework practices and to study mothers as they helped their 5th grade children with challenging mathematics problems.

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1. Homework

The research reported here weaves together strands of prior research involving the importance of children's homework and parental involvement with homework. A number of studies point to the importance of homework in students' mathematics achievement. An analysis using the National Education Longitudinal Studies (NELS88) data indicated that extra time spent on mathematics homework increases students' mathematics test scores (Aksoy & Link, 2000). The effect of homework time on test performance was large, suggesting that improvement in this area could produce substantial gains in performance. Paralleling well-documented differences in mathematics achievement between Asian students and American students (Stevenson et al., 1993; Stevenson, Lee, & Stigler, 1986), Chinese and Japanese children spend more time on homework than American children do (Chen & Stevenson, 1989; Fuligni & Stevenson, 1995). Stevenson has also argued that part of the math gap between American and Asian students is due to cultural differences in attributions for performance. American students and mothers believe that innate intelligence is a major factor in mathematics performance and tend to dismiss effort as a factor, whereas the pattern for Asians is just the opposite (Stevenson et al., 1993; see also Li, 2005). Working on homework, of course, requires effort.

The merits of homework and the links between homework and students' achievement, however, are complex and debated (Corno, 1996, 2000; Xu & Corno, 1998, 2003). One study demonstrated that the frequency of homework assignments had a positive effect on gains in mathematics achievement, but length of homework assignments had a negative effect for some students (Trautwein, Köller, Schmitz, & Baumert, 2002). Certainly there is a consensus that homework assignments must be designed well in order to be effective (Epstein & Van Voorhis, 2001).

2. The quality of parental involvement in homework

Research provides evidence of the importance of parental involvement with homework in students' school achievement (Gutman & Eccles, 1999). Most of the research on parent involvement, though, has used a relatively crude measure of the *quantity* of involvement, self-reported on a rating scale or in minutes per week (e.g., Levin et al., 1997). Yet there is good reason to think that the *quality* of parental involvement is at least as important as the quantity, when the outcome is the child's academic achievement. A few studies, outside the domain of mathematics, provide suggestive evidence. For example, Conger, Conger, and Elder (1997) found that mothers' harsh, inconsistent parenting negatively affected adolescents' self-confidence, which in turn had a negative effect on grades. Cooper and Lindsay (2000), aggregating across all school subjects, found that parental self-reported support for the child's autonomy in doing homework was associated with higher standardized test scores and better grades (see also Cooper, 1989). Parental involvement with homework may have a negative effect, for example, if parents are ill-prepared in the subject or if they become overly involved, perhaps even completing assignments themselves (Cooper, 1989). Parental disapproval during instruction is associated with worse performance by the child at a later time (Gauvain, Fagot, Leve, & Kavanagh, 2002).

3. Theoretical frameworks

The research reported here rests broadly on Vygotsky's theory (1978) and its extension by Rogoff (1990). Vygotsky's most basic premise was that the child's intellectual development is heavily rooted in the child's social environment; he suggested that the social unit is primary for the child and that higher cognitive processing grows out of it. Vygotsky argued that cognitive development occurs when an adult structures and models solutions to problems, in what Vygotsky called the zone of proximal development (ZPD) (an area in which the child cannot yet quite solve a problem independently and is responsive to social guidance). Rogoff (1990) referred to this socially structured and facilitated cognitive development as "apprenticeship in thinking." The Vygotskian view is clearly linked to our focus on the importance of parent–child interactions that provide tutoring or apprenticeship in children's mathematics learning.

An important aspect of the quality of parents' approach to helping with mathematics problems is scaffolding (Vygotsky, 1978; Wood & Middleton, 1975). In scaffolding, the adult is sensitive to the needs of the learner, identifying the child's level of knowledge in relation to the problem to be solved. Scaffolding is effective, according to Vygostky, because the adult is teaching within the child's zone of proximal development, that is, slightly beyond the child's level of competence. Aspects of scaffolding include recruiting the child's interest in the task, simplifying the task and reducing the number of steps so that the child can manage smaller components, encouraging continued motivation, pointing out cases where the child's solution does not match the ideal solution, controlling the child's frustration, and demonstrating how to solve the problem (Rogoff, 1990). The adult then readjusts his or her approach as the child progresses to a new level of competence.

Scaffolding by teachers in the classroom has been studied, both in non-mathematical areas (Chi, Siler, Jeong, Yamauchi, & Hausmann, 2001; Fleer, 1992; Hacker & Tenent, 2002) and in mathematics (Meyer & Turner, 2002; Nathan & Knuth, 2003; Sarama, Clements, Swaminathan, McMillen, & Gonzalez-Gomez, 2003). Only a few studies have examined scaffolding by parents, mostly in non-mathematical domains (Evans, Moretti, Shaw, & Fox, 2003; Neitzel & Stright, 2003) but also in mathematics (Conner & Cross, 2003; Pratt, Green, MacVicar, & Bountrogianni, 1992; Shumow, 1998). Results from these studies generally show that the quality of scaffolding is positively related to the child's subsequent performance (e.g., Conner & Cross, 2003).

4. The current study

The purpose of the current research was to study mathematics homework practices and to investigate mother–child interactions while working challenging mathematics problems. In the domain of quantity, we assessed the amount of time the child spends, on average, on mathematics homework, and the amount of time the child receives help from a parent or other skilled person. To assess the quality of interactions, we used the Math Homework Task (MHT), a structured task designed to serve as an analog to natural situations in the home in which parents help children with mathematics homework. For these children, who had just completed 5th grade, we used pre-algebra equivalence problems of the type $3+4+5=3+\_\_\_\_\_$. These problems were chosen because (a) they are an important stepping stone in making the crucial transition to algebra (Capenter, Franke, & Levi, 2003; Kieran, 1981, 1992; Knuth, Alibali, Weinberg, McNeil, & Stephens, 2005); (b) typically they have not yet been encountered by 5th graders; and (c) they should be within the zone of proximal development for most 5th graders. The interactions, which occurred in the home, were videotaped and later coded for two broad areas of content: (a) mathematical content (e.g., did the mother explicitly discuss the equal sign?); and (b) the mother's scaffolding (e.g., did the mother match her teaching to the child's zone of proximal development?). We studied mother–child interactions exclusively, and not father–child interactions, in order to hold constant the gender of the parent and because past research has shown that mothers are the chief source of homework help, even in areas such as mathematics that traditionally are male stereotyped (Crystal & Stevenson, 1991). We also investigated what factors were linked to the best performance by mothers, focusing on mothers' education, mathematics preparation, and mathematics self-confidence.

5. Method

5.1. Participants

Although data were collected from 165 mother–child dyads, equipment malfunctions made seven of the videotapes uncodeable. Therefore, participants were 158 children (76 girls, 82 boys) who had just completed 5th grade, and their mothers. The mean age of the children was 11.4 years (S.D. = 0.19). These children comprise Cohort 2 of the children in an ongoing longitudinal project, the Wisconsin Study of Families and Work (WSFW; Hyde, Klein, Essex, & Clark, 1995). The WSFW sample was originally recruited in 1990–1991 for a study of parents balancing work and family at the time of the birth of a child; the families continue to be followed longitudinally. In regard to ethnicity, 146 (92%) of the children were White (not Hispanic), 6 (4%) were Black, 4 (3%) were Native American, and 2 (1%) were Hispanic. This ethnic distribution reflects that of the state in which the data were collected. Approximately 80% came from the Milwaukee area and 20% from the Dane County, Wisconsin area. The children attended a wide variety of elementary schools, mostly in southern Wisconsin. Mothers' education averaged 15 years (S.D. = 2.1) on a scale on which high school graduate was equivalent to 12 years and college graduate was equivalent to 16 years; 1% of mothers had not completed high school, 16% had graduated from high school, 10% had attended technical school beyond high school, 18% had some college, 35% had graduated from college, and 22% had education beyond college. All mothers had English as their first language or spoke and read it well enough to complete questionnaires and interviews in English.

5.2. Procedure

Two interviewers administered the tasks in the family's home. One worked with the child, administered questionnaires to the child, and administered the MHT to the mother and child. The other began in a separate room with the mother, administered questionnaires, briefed her on the MHT, and operated the video camera during the MHT.

Before the teaching segment of the MHT began, the mother was briefly introduced to the class of problems that she would be helping her child to learn. This briefing served two purposes: it ensured that all mothers had at least some basic knowledge of what to expect, and it served as an analog to potential school–family partnerships in which parents might receive information about the types of mathematics problems their children would be working that week in school. To ensure that mothers would have available both procedural and conceptual strategies (Rittle-Johnson & Alibali, 1999; Silver, 1986), the mother was told about both a procedural strategy and a conceptual approach for the problems. Specifically, the mother was told:

> We'd like you to teach your child how to do a type of math problem called equivalency problems. They're really pre-algebra problems. [Mother is shown a laminated card with examples.] Most kids [name]'s age haven't learned how to do these problems in school yet. We've found that it helps children understand these problems if you focus on the equal sign and the idea that the two sides of the equation must be equal to each other. This card shows an equivalency problem and a canceling strategy that may help your child solve these. [Interviewer talks through card with mother. The card includes a problem with the same number on both sides of the equation so that it can be cancelled.] You may teach the canceling strategy, or you and [name] may solve the problems in other ways.

The canceling strategy represents a procedural approach. The emphasis on the equal sign and the two sides of the equation is a conceptual approach.

The mother and child were then reunited. The child was given a baseline problem, $4+3+8=3+$ ______, worked without the mother's help, to determine whether the child was already proficient at this type of problem. The child was given 30 s to solve the baseline problem.

The mother and child then had 5 min together for teaching and learning. To assist in this interaction, dyads were given a packet of 19 practice problems, printed one at the top of each page. They increased in difficulty so that a simple canceling strategy did not work by the third problem, although it could be solved by canceling $2+3$ on one side and 5 on the other. By the fifth problem, no canceling strategy worked ($8+2+4=9+$ ______). Problem 13 shifted to multiplication ($3\times 2\times 4=3\times$ ______) and problem 15 shifted to mixed operations ($2\times 3\times 4=$ ______ $+10$). Five minutes was chosen for the length of the interaction because our pilot data showed that the amount of homework help provided by parents per day is often in this range.

5.3. Measures

5.3.1. Baseline problem

Children's answer to a baseline problem, administered before the mother's teaching time, was recorded. The baseline problem was $4+3+8=3+$ ______ .

5.3.2. Mathematical content of mother's lesson presentation

Six variables were coded from videotapes of the teaching segment. The first two variables were based on codes developed for teachers' lesson presentations by Romberg, Shafer, and Webb (2000).

1. Level of mother's lesson presentation: no formal presentation (1), procedural instruction only (2), some conceptual teaching (3), strong emphasis on conceptual understanding (4).
2. Mother's teaching of multiple strategies to solve the problems: no strategies taught or none taught clearly (1), teaches a single strategy only (2), some emphasis on multiple strategies (3), strong emphasis on multiple strategies (4).
3. Mother spoke of equal sign: mother did not mention it (1), mother mentioned it once (2), mother mentioned it more than once (3).
4. Mother spoke of sides of the equation or sides: mother did not mention it (1), mother mentioned it once (2), mother mentioned it more than once (3).
5. Mother taught canceling procedure: no (1), yes (2).

An additional variable was created, a single score representing the overall quality of the mathematics content of the mother's presentation; this mathematics composite score was formed for each mother by converting scores on level of lesson presentation, teaches multiple strategies, equal sign, and sides of the equation to z scores, and averaging the z scores. The variable taught canceling procedure was not included because it is ambiguous whether it reflects high-quality teaching. Teaching the canceling procedure is better than nothing, but it focuses only on a procedure while ignoring important concepts.

5.3.3. Mother's scaffolding

Based on the work of Rogoff (1990; see also Pratt et al., 1992), we defined and coded the following scaffolding variables:

1. Recruits the child's interest: no attempt to recruit interest (1), minimal attempt (2), moderate attempts and shows interest herself (3), considerable effort in recruiting interest (4), considerable effort in recruiting interest and states that the task is important or interesting, or equivalents such as "cool" or useful (5).
2. Simplifies the task to manageable, small components: no attempt to simplify, provides too much information at once (1), broad overview with little attention to smaller components (2), explains one component clearly, e.g., equal sign, but does not go beyond that (3), explains two or three components clearly but could be more logical (4), successfully simplifies the task into manageable components and explains them in a logical fashion (5).
3. Sustains the child's motivation: mother behaves in ways that discourage the child (1), mother occasionally or mildly discourages the child (2), mother is neutral with respect to motivating (3), mother actively encourages the child, but not always (4), mother shows high level of actively encouraging the child and herself shows sustained enthusiasm (5).
4. Notes errors or imperfect solutions: never points out errors (1), points out error once but offers inadequate explanation (2), points out error more than once and offers inadequate explanation (3), points out error more than once and offers some explanation (4), consistently points out errors and offers clear explanation, asks hinting questions, or asks the child to explain why it was wrong (5).
5. Controls child's frustration: mother's behavior increases the child's frustration substantially (1), mother's behavior increases the child's frustration slightly (2), mother does nothing, positive or negative, in relation to child's frustration (3), mother acknowledges the child's frustration in a limited way and provides some encouragement (4), mother is calm and patient and effectively controls the child's frustration (5). If the child displayed no frustration, the variable was coded 9. In contrast to the 3rd graders in our pilot study, few of these 5th graders displayed frustration during the teaching segment, so most mothers were not rated on their efforts to control the child's frustration. This variable is therefore not included in any subsequent analyses.
6. Demonstrates how to solve the problem: does not demonstrate how to solve a problem (1), provides quick or partial demonstration that is unclear and ineffective (2), demonstrates how to solve the problem fairly well but is slightly unclear (3), demonstrates how to solve the problem well, at a level and speed appropriate to the child (4). Note that this variable has a range of only 1–4.
7. Matches teaching to child's zone of proximal development: no effort to identify child's level of understanding (1), some attempt to identify child's level but is unsuccessful and teaches at a level too high or too low for the child (2), identifies child's level fairly well and generally teaches at the appropriate level (3), quickly and sensitively identifies child's level and adapts her teaching well to the child's level (4). Note that this scale has a range of only 1–4.
8. Transfers responsibility to child at appropriate time: transfers responsibility much too early (1), transfers responsibility a bit too early (2), transfers responsibility at the appropriate time, i.e., when the child appears to have mastered the task and is ready to proceed independently (3), transfers responsibility a bit too late (4), transfers responsibility much too late (5).
9. Displays interest: shows obvious lack of interest in the task or is neutral (1), shows interest inconsistently (2), shows moderate interest (3), shows great interest in and enthusiasm for the task (4). Note that this variable has a range of only 1–4.

A composite scaffolding score was computed for each mother by first converting each mother's score on each variable (excluding controlling child's frustration) to a z score, and then averaging z scores on all eight variables. Before doing this, however, the variable "transfers responsibility at appropriate time" was transformed, because for that variable the optimal score was the middle of the scale. Therefore, scores of 1 or 5 (transferring responsibility much too early or much too late) were coded 1, scores of 2 or 4 (transfers responsibility a bit too early or late) were coded 2, and scores of 3 (transfers responsibility at appropriate time) were coded 3.

5.3.4. Mother's level of mathematics preparation

On a written questionnaire, the mother reported her own mathematics preparation from among these alternatives: less than high school algebra, high school algebra, high school geometry or trigonometry, one year of calculus, and courses beyond calculus. She rated how good she was at math on a scale from 1 (*no good at all*) to 7 (*very good*), and her confidence in helping her child with math homework on a scale from 1 (*not at all confident*) to 7 (*very confident*).

5.3.5. Mathematics homework practices

As part of a written questionnaire, mothers reported on a number of homework practices in their home. They reported which parent helps more with math homework on a scale from 1 (*dad does all the helping*), to 3 (*we help equally)*, to 5 (*I do it all).* Alternatives of *neither parent helps*, and *teacher does not send homework home* were also offered. Another question asked, "On an average school day, how many minutes does your child spend at home doing math homework?" This was followed by a question in which they allocated those minutes into minutes the mother helps, minutes the father helps, minutes the child works independently, and minutes someone else (such as an older sibling) helps. Mothers also responded, in writing, to the following open-ended question: "What is your goal or approach when you help your child with math homework?" Responses to this last question were content-analyzed.

5.3.6. Influences on students' mathematics performance

Following Stevenson et al. (1993), we asked mothers to rank the following four factors that might influence students' performance in mathematics: good teacher, innate intelligence, home environment, and studying hard.

5.3.7. Videotape coding

Coders were advanced undergraduates who received more than 50 h of training, which included reading Vygotsky (1978) and Rogoff (1990); coding a similar task videotaped with 3rd grade children and their mothers until they reached strong reliability; and class instruction led by the first author, on six videotapes selected to represent a variety of interactions. For mathematical variables, 138 of the 158 videotapes were coded independently by two coders. Interrater reliability was as follows: 91% agreement for answer to baseline problem; 87% for number of practice problems completed; 65% for level of lesson presentation (and 98% agreement within 1 scale point); 71% for mother's teaching of multiple strategies (98% within 1 point); 82% for mention of equal sign; 85% for mention of sides of the equation; and 97% for teaching the canceling procedure. For scaffolding variables, 136 of the 158 videotapes were coded independently by two coders. Interrater reliability was as follows: 63% for recruiting the child's interest (100% for agreement within 1 scale point); 55% for simplifying the task (85% within 1 point); 74% for sustaining motivation; 60% for noting imperfect solutions (85% within 1 point); 83% for controlling the child's frustration; 55% for demonstrating how to solve the problems (93% within 1 point); 60% for matching teaching to child's ZPD (98% within 1 point); 75% for transferring responsibility (100% within 1 point); and 68% for mother's interest (95% within 1 point). In all cases of double coding, discrepancies were discussed by the coders and resolved by consensus. Consensus codes have higher validity than individual raters' codes (Uebersax, 1988).

6. Results

6.1. Reports of homework practices

In regard to the quantity of mathematics homework, mothers reported that their child, on average, spent 23 min per day on mathematics homework (range 0–80). During that time, mothers provided an average of 4 min of help and fathers provided and average of 4 min. On the 5-point scale reporting whether father helped more or mother did, there was a fairly even distribution and much sharing of responsibility; 7% reported that dad does all the helping, 22% that dad did most of the helping, 25% that both parents helped about equally, 20% that mom did most of the helping, and 17% that mom did all the helping. An additional 6% reported that neither parent helps, and 4% that the teacher does not send home math homework.

When asked to evaluate the amount of math homework their child received, most mothers (72%) felt that it was about right, 21% felt there was not enough, and only 7% thought there was too much.

6.2. Influences on students' mathematics performance

Teachers ranked first in mothers' estimates of factors influencing students' mathematics performance; 47% of mothers ranked teachers first (mean rank 1.73). Innate intelligence was also ranked as an important factor ($M = 2.13$); it was chosen as the most important factor by 43% of mothers. Studying hard ranked a distant third ($M = 2.93$), with 5% of mothers ranking it as the most important factor, and home environment ranked fourth ($M = 3.22$), with 6% of mothers ranking it as most important.

6.3. Mothers' goals in helping with mathematics

Mothers' responses to the open-ended question "What is your goal or approach when you help your child with math homework?" were content-analyzed. Table 1 displays the categories that emerged and the percentage of mothers mentioning each goal. Developing the child's understanding and competence with mathematics problems and helping the child to become independent in solving problems were the two most common goals.

6.4. MHT: baseline problem and practice problems

Unexpectedly, 48% of the children gave the correct answer, 12, on the baseline problem, $4 + 3 + 8 = 3 + \_\_\_\_$. The other common answer was 18 (given by 29% of children), which results from adding through all numbers, treating the = as if it were a +. The remaining children gave a variety of incorrect answers ranging from 8 to 19. On average, during the 5-min teaching segment, children completed 11 problems from the packet of 19 practice problems (range 0–19).

We noticed that when children gave the correct answer on the baseline problem and showed that they understood how to work these problems, mothers did much less by way of mathematical presentation and scaffolding. In a sense, they matched their teaching to the child's ZPD, but it also meant that they had little to teach in the teaching task. To test our hypothesis, we compared mothers of children who were correct (CC) on the baseline problem ($n = 76$) with mothers of children who were incorrect (CI; $n = 83$) on mathematical content (in one MANOVA, two groups, five dependent variables) and scaffolding (in a second MANOVA, two groups, eight dependent variables). The results indicated significant differences between CC mothers and CI mothers in mathematical content, multivariate $F(5, 148) = 12.27$, $P < .0001$. CI mothers gave higher-level lesson presentations, were more likely to teach multiple strategies, were more likely to mention the equal sign, were more likely to teach the canceling procedure, and were more likely to discuss sides of the equation. The comparison of CC and CI mothers on scaffolding variables was also significant, multivariate F(8,

Table 1
Mothers' goals in helping with mathematics homework

Mothers' goals	Percentage of mothers reporting the goal
Competence: Wants child to develop understanding and competence with problems	48
Independence: Wants child to develop ability to solve problems independently	28
Confidence: Build child's confidence in doing math	4
Real-life applications: Convey to child the practical importance of math	6
Get the job done accurately	15
Be available if child has questions or encounters difficulties	14
Math is fun: Convey that math can be enjoyable	5
Positive interaction: Keep the interaction positive	2
Avoid conflict	7
Learn with the child	4
No help because child doesn't need it	2
No help because child never asks for it	1
No help because the material is too advanced for the mother	2

Note. Percentages do not sum to 100 because some mothers listed more than one goal and some mothers listed none.

119) = 3.31, $P < .01$. CI mothers scored higher on recruiting interest, simplifying the task, noting imperfect solutions, and demonstrating how to solve the problem; there were no group differences on the other scaffolding variables. Essentially CI mothers tried harder. This also meant, though, that the teaching segment was not a true teaching segment for CC mothers, whose children already understood these problems. CC children, for example, completed more practice problems ($M = 13.8$) than CI children did ($M = 9.2$), $F(1, 155) = 42.96$, $P < .001$. CC children also were correct on 50% more practice problems ($M = 12.3$) than CI children ($M = 8.2$), $F(1, 152) = 36.70$, $P < .0001$. We therefore excluded CC dyads from the analyses that follow, and focus on CI dyads, for whom the situation provided opportunities for scaffolding.

6.5. Mathematical content of mother's lesson presentation

We assessed the quality of the mother–child interactions in terms of both mathematical content and scaffolding. The results revealed great diversity in the mathematical content of mothers' presentations. Ten percent did not mention the equal sign, 29% mentioned it once, and 61% mentioned it more than once. Sixteen percent did not use the term "sides of the equation" or "side," 12% did so once, and 72% did so more than once. Ninety-six percent of the mothers taught the canceling procedure (or taught "crossing out" or canceling using fingers rather than pencil marks).

In regard to the level of lesson presentation, 5% of mothers provided no formal presentation; 11% demonstrated a procedure such as canceling but did not go beyond that; 63% of mothers attempted to develop some conceptual understanding; and 21% strongly emphasized conceptual understanding. In regard to the teaching of multiple strategies for solving the problems, 5% of mothers taught no strategy or presented a strategy unclearly; 12% taught a single strategy only, such as canceling matching numbers on both sides of the equation; 68% placed some emphasis on multiple strategies, explaining positively that there are two or more ways to solve the problems; and 15% placed strong emphasis on multiple strategies.

To convey a bit more about the nature of these interactions, extracts from transcripts are offered as illustrations. These extracts, of course, convey the verbal content and only limited information about other qualities such as gestures, but they do convey much of what the conversations were like. The following is an extract from a mother who scored high on the mathematical content of her presentation as indicated by her math composite score. The child has just gotten the baseline problem, $4 + 3 + 8 = 3 + \underline{\qquad}$ wrong, answering 9. This mother, in response, goes back to the problem to clarify it for her daughter.

1	Mother (M):	All right, should we start with this one? [Pointing to baseline problem]
2	Child (C):	Yea, let's start with this one. [Erases incorrect answer]
3	M:	This is equals [points to the equal sign]. So think about the two sides of the equals have to be . . .? [Gesturing to two sides]
4	C:	Equal — Oh! OK!
5	M:	Equal. So 4 plus 3 is . . .
6	C:	7
7	M:	Plus 8 . . .
8	C:	Is tw — oh not 12. [Laughs, looks at mom]
9	M:	Is that what you were thinking when you put a 9 down?
10	C:	Yeah.
11	M:	OK, so what's the 4 plus 3?
12	C:	What?
13	M:	What does this much equal? [Covers the right side of the equation with her hand, leaving the left side
14		visible]
15	C:	Equals [pauses hits forehead] . . .
16	M:	Honey, what's 4 plus 3 [points]
17	C:	15. Oh, 7! OK!
18	M:	15
19	C:	15
20	M:	OK, so this side equals 15 [points to left side], so 3 plus what equals 15?
21	C:	OK, so that would be 12 [writes 12 in blank].
22	M:	OK.
23	C:	Right.
24	M:	Then there's another thing they showed me that you can do to make these easier. Is like, on the equal
25		sides [gestures with both hands like balance scales] these would cancel out [covers 3s with her fingers].
26		If you have a 3 on this side and a 3 on this side they cancel out. So you just would add 4 plus 8 and that's

[points to earlier answer, 12].
C: Oh, OK.
M: 12
C: [laughs]
M: So if you have the same number on either side of the equation it kinda cancels each other out, then you add up the ones that are left.
C: OK. [Turns page]
[They move on to 15 + 7 + 10 = ________ + 10.]
M: OK? So let's try this one.
C: All right, so . . .
M: Do you want to try doing the cancel out thing?
C: Um no, 'cuz it looks different. [Laughs]
M: Well there's the two numbers on the same side . . .
C: So you take this [covers up 10s on both sides] . . .
M: You take the two that are the same, you just don't count those. And then what's . . .?
C: 22
M: Yep. [Nods]
C: [writes answer, 22, in blank] OK.
M: So if you add that [points to left side of equation], so this, 15 and 10 is?
C: 25
M: Plus 7 is
C: Plus 7 is 32.
M: 22 plus 10 is?
C: Uh, 32.
M: Yep, that's how you can check it.
[They move on to 7 + 11 + 16 = 11 + ________]
C: Let's look at the new one.
M: OK.
C: Um, so we take these out [covers the 11s]
M: Mmhmm. [Nods]
C: And then you do 16 plus 7? [Looks at mom]
M: [nods] Mmhmm.
C: OK, 23?
M: Yeah. And then you can check it to see if it's right.
C: Do I . . .?
M: By just adding up [points to left side] and adding up [points to right side].
C: 33 and 3 — uh OK. [Laughs] 23, 33, 34. This is 34 and 11, 16, 22 . . . [having trouble adding the numbers in her head]
M: You can do scratch paper if it would help you. If you want to make a row or whatever like you do at school. Want to do it in your head?
C: OK, I'm going to say it equals 34 'cuz I got 35 [laughs]
M: Why don't you just check it on scratch paper? Just write it out like you do at school.
C: [writes in column on scratch paper to add] Yes 15, OK.
M: And yep, OK.
C: And then, so that's that?
[They move on to 2 + 3 + 7 = 5 + ________]
C: OK, next one. 6.
M: Right
C: Oh, 7. [Laughs, writes 7]
M: Try it and see if it works.
C: 7 plus 5 is 12. And then 7 plus 3 is 10, 11, 12. [Looks at mom]
M: [nods]
C: [Turns page. They move on to 4 + 5 + 7 = 9 + ________]
M: Um, you know what? Should I show you another way you might cancel this?
C: OK.
M: To cancel it would be easier. This is what? This is 9. [Points to 9 on right side]
C: 9
M: Do any of these numbers add up to 9? [Points to left side]
C: Yeah.
M: These two, right? [points to 4 + 5] So you cancel that [covers 4 + 5] and that cancels [covers 9 on right]
C: 7?
M: So then there's your answer.

90 C: OK. [Writes it in blank.]
91 M: Let's see if you can — that might make it easier too.
92 C: [Turns page. They are now on $8+2+4=9+$ ______]
93 M: Oh sure, on this one we can't do that.
94 C: [laughs] OK, um 14 [adding left] . . . 5?
95 M: Mmhmm.
96 C: [Turns page. They are now on $9+6+2=3+$ ______]
97 C: 17, so that would be 14, right?
98 M: Mmhmm.
99 C: [Turns page. They are now on $5+6+7=$ ______ $+4$]
100 C: 18 [adding left side], mmm, 14? [Looks at mom]
101 M: [nods]
102 C: Right?
103 M: Yeah.
104 C: [laughs, writes the 14 in blank]
105 M: You know. You got it.

This mother first used a conceptual approach in which she explained the meaning of the equal sign and the principle that the sides of the equation must be equal (Line 3 ff). Later she introduced the shortcut procedural strategy of canceling (Line 25 ff). She taught multiple strategies including checking the work by adding up each side of the equation, which reinforced the principle of the equivalence of the two sides of the equation (Line 46 and again in Line 61). By the fifth practice problem the child seems to have mastered the concept and consistently displayed mastery in succeeding problems. This mother had completed one year of calculus and held an advanced degree beyond college.

The following excerpt offers a sharp contrast, with a mother who scored very low on mathematical content as indicated by her math composite score. The mother clearly does not understand the principal of equivalence herself, and thinks that canceling involves eliminating numbers in the same position in the equation, rather than numbers with the same value. They begin by reworking the baseline problem, $4+3+8=3+$ ______ , which the daughter had gotten wrong, answering 18.

1 M: OK, why don't you erase it. [Child erases wrong answer]
2 [Pause. Mother looks at help card.]
3 C: [talking to self] Ohhh, 21.
4 M: [ignoring child's comment] OK. There's a way that you can work these problems a
5 little easier. It's algebra and it's called the canceling strategy. If you take the first
6 number and put a line through it [child cancels the 4] and you take the last number and
7 put a line through it [child cancels the 3 on the right side] and if you add the first . . . these
8 two numbers together, what do they make?
9 C: 11
10 M: OK, you can put that down here [in the blank] so you can remember that. And if
11 you . . . if you . . .
12 C: Add those two? [Pointing to $4+3$]
13 M: Mhmm.
14 C: 7
15 M: Well, wait a minute. It's actually um, if you add all thr . . . add all three of these up
16 together.
17 C: Those? [Gesturing to left side of equation]
18 M: Mhmm.
19 C: 15.
20 M: OK. You take away the 4. [Unintelligible] So, the answer is actually 11. [Child writes
21 answer.] It's very, it's very confusing.
22 [They move on to $15+7+10=$ ______ $+10$.]
23 M: 15 plus 7?
24 C: is 22
25 M: plus 10
26 C: 32
27 M: OK, let's try the canceling out. OK, this is a little bit different, because this isn't . . .
28 this is the answer right here. OK? So you have to find what number would be, for the
29 equals number. OK? So basically you know that this number, 15, after that . . . how this
30 one is . . . [begins to flip to previous page]
31 C: Wait, I think I have it. 15 take away 10 is 5 . . .

M: 15 plus . . .
C: And then 5 take away um 7 is negative 2 and then plus . . . what?
M: Go ahead.
C: Plus 10, is 12.
M: [flips to previous page]
C: [flips back] That's confusing. OK. [Counts on fingers]
M: 15 plus 7 . . . 25. OK, we'll use this to help us. [Looks back at help card] This number [underlines 15 on left side] and this number [underlines 10 on right]. This isn't the answer [pointing to the 10 on the right side]. So, they're looking for what number. Oh, they're looking for what this number is. [Points to blank.] Which is 15. Cause you add the two numbers. You add these two numbers, that's 17. And then you take. You take um, what did she say, the 15? Take those two out. This is actually our answer, these two numbers. So they're looking for this. [Mother writes 15 on space.] 15. So you take 15, so 17.
C: [turns page]
[They move on to $7 + 11 + 16 = 11 +$ ________]
M: I'm not sure if that was right. OK, so you take this one out [7 on left side], and you take the 11 out [on right side], and you add these two together [pointing to 11 + 16]. Add these two together, and write it down.
C: 27 [writes down]
M: 27. OK.

This mother clearly does not understand the concept of equivalence herself. She provides no conceptual instruction and instead focuses immediately on procedural information, canceling, which she teaches incorrectly. The child at one point comes close to understanding the concept herself (Line 31: Wait, I think I have it. 15 take away 10 is 5 . . .), but her mother interrupts with confusing information and takes her away from a potentially correct solution. This mother was a high school graduate and reported having completed less than high school algebra.

6.6. *Mother's scaffolding*

Table 2 displays the distribution of CI mothers' scores on the nine scaffolding variables. Overall, there was considerable variability in mothers' behaviors. In regard to recruiting the child's interest, most mothers made only moderate attempts. Most mothers did a good or excellent job at simplifying the task into manageable components, but 17% showed weak performance as indicated by ratings of 3 or less. In regard to sustaining the child's motivation, most mothers (64%) displayed strong efforts as indicated by ratings of 4 or 5, but 31% did little to sustain motivation, and 4% behaved in ways that would actually discourage the child. The following extract illustrates a mother sustaining motivation, after the child has just completed a problem correctly:

M: Perfect! You'll love algebra! I'm telling you — it's just too much fun!

Table 2
Mothers' scaffolding during the teaching session

Scaffolding variable	Score					
	1	2	3	4	5	9
Recruits the child's interest	0	9%	68%	24%	0	3%
Simplifies the task to manageable components	2%	2%	13%	42%	40%	3%
Sustains child's motivation	2%	2%	31%	46%	18%	2%
Notes imperfect solutions	4%	3%	7%	49%	38%	
Controls the child's frustration	0	5%	5%	6%	4%	80%
Demonstrates how to solve problem	1%	14%	38%	47%	NC	3%
Matches teaching to child's ZPD	5%	6%	45%	44%	NC	2%
Transfers responsibility at appropriate time	2%	6%	73%	17%	1%	1%
Displays interest	0%	7%	63%	29%	NC	1%

Note. $n = 83$. See text for definitions of scores on each variable. 9 = not applicable or uncodeable. NC = code of 5 not available for that variable.

When children made errors, almost all CI mothers called it to their attention (Table 2). Forty-seven percent of the mothers demonstrated how to solve these problems well and an additional 38% did so fairly well but were slightly unclear, went too fast, or were not quite matched to the child's level; 1% never demonstrated how to solve these problems and 14% provided an ineffective demonstration. Most mothers matched their teaching to the child's ZPD, and 73% transferred responsibility to the child at the appropriate time, not too early or too late. In regard to interest, 29% displayed great interest and enthusiasm and an additional 63% demonstrated moderate interest.

In the following extract, the mother identifies the child's ZPD, simplifies the task, and provides exactly what the child needs, a discussion of sides of the equation and the equal sign. The child has just gotten the baseline problem, $4+3+8=3+\_\_\_\_\_\_$, wrong, answering 18.

M: OK, can you explain to me what you did? How did you come up with 18?
C: I just added 'em.
M: How did you add them?
C: That was 15 plus 3 was 18.
M: With the equal sign, right?
C: It's 15 plus 3 equals 18.
M: OK, how these problems work is that these two numbers [M points] on this side have to equal these numbers on this side. OK, this is an equal sign. So if this is 15, then this has to equal 15. On both sides, you see?

Returning to the first long extract to consider the mother's scaffolding, we see that she simplified the task by covering the right side of the equation so the child could focus on computing the value of the left side (Line 14 ff). At Line 56 she begins to transfer responsibility, letting the child take the lead in solving the problem.

In the second long extract, the mother's scaffolding is poor because she does not herself understand how to solve the problems. Although she attempts to demonstrate how to solve the problem, she is never successful in doing so. She is unable to transfer responsibility to the child at the appropriate time because the child makes no progress in understanding and the mother is unable to assess whether the child is beginning to understand. The mother fails to sensitively identify the child's ZPD (she does not spot how the child got 18 and the misunderstanding it represents) and ignores several of the child's significant comments (e.g., Line 31).

6.7. Mothers' education and mathematics preparation and self-confidence

Mothers on average had completed high school geometry or trigonometry, but there was considerable variability in preparation; 6% had completed less than high school algebra, 12% had completed high school algebra, 51% had completed high school geometry or trigonometry, 18% had completed one year of calculus, and 13% had completed courses beyond calculus.

On the 1–7 scale of math self-confidence, mothers averaged a self-rating of 4.78. Again, there was considerable variability, with 3% of mothers answering 1 (not good at all) and 12% answering 7 (very good). In regard to confidence in helping her child with math homework, mothers on averaged reported 4.89, or slightly above the mid-point, on the 7-point scale. At the extremes, 4% rated themselves as not at all confident and 18% rated themselves as very confident at helping with math homework.

Table 3 shows correlations between features of the mother's lesson presentation and her own education, preparation in mathematics, and self-confidence in mathematics, for CI mothers only. Notice that the correlation between the mother's math composite teaching score and her mathematics preparation was $r=.23$, $P<.05$. Mother's math preparation showed significant correlations specifically with teaching multiple strategies and discussing sides of the equation. The correlation between the mother's composite scaffolding score and her own mathematics preparation was $r=.27$, $P<.01$. Mothers with more mathematics preparation were better at simplifying the task to manageable components and noting imperfect solutions. Correlations for all other scaffolding variables were in the predicted direction but did not quite reach significance.

Mother's math self-confidence was related to her math composite teaching score, $r=.38$, $P<.01$. Mothers with more self-confidence presented higher-level lessons, were more likely to teach multiple strategies, and were more likely to discuss sides of the equation. Mothers' math self-confidence did not correlate significantly with

Table 3
Correlations between mothers' scaffolding and mathematics content and mothers' education, mathematics preparation and confidence (for children who were incorrect on the baseline problem)

	Mother's mathematics preparation	Mother's math self-confidence	Mother's education
Mathematical content			
Math composite	.23*	.38**	.29**
Level of lesson presentation	.14	.22*	.33**
Teaches multiple strategies	.29**	.30**	.30**
Canceling procedure	.09	.17	.03
Equal sign	.03	.25*	.05
Sides of the equation	.23*	.37**	.17
Scaffolding			
Scaffolding composite	.27**	.19	.33**
Recruits the child's interest	.15	.02	.13
Simplifies task to manageable components	.24*	.25*	.25*
Sustains child's motivation	.14	.04	.25*
Notes imperfect solutions	.26*	.11	.31**
Demonstrates how to solve problem	.18	.24*	.21
Matches teaching to child's ZPD	.21	.19	.30**
Transfers responsibility at appropriate time	.15	.13	.07
Displays interest	.21	.14	.29**

Note. $n = 83$. Math composite is an average of the mother's scores on level of lesson presentation, teaches multiple strategies, equal sign, and sides of the equation.
* $P < .05$.
** $P < .01$.

the scaffolding composite, but did correlate significantly with simplifying the task to manageable components and demonstrating how to solve the problem. Mothers' preparation correlated $r = .40$, $P < .01$, with their math self-confidence.

Interestingly, mother's education generally correlated somewhat more strongly with the quality of her scaffolding, and displayed more significant relationships, than the mother's mathematics preparation did (Table 3). Compared with mothers with less education, mothers with more education were better at matching their teaching to the child's ZPD and displaying interest and enthusiasm. Surprisingly, mother's education correlated only $r = .26$, $P < .05$, with her mathematics preparation. Some mothers managed to pursue quite a bit of education that did not include additional mathematics courses.

7. Discussion

The overall goal of this research was to study mathematics homework practices and mothers' interactions with their 5th grade children as they worked on mathematics together in a task designed to serve as an analog to naturally occurring homework situations. The results revealed considerable diversity among mothers in this situation; some were excellent teachers and others displayed deficiencies that might be remedied by school–family partnerships. For example, even following prompting, 10% of mothers did not mention the equal sign, 16% did not discuss sides of the equation, and 16% did not attempt to develop conceptual understanding. Many mothers were excellent at scaffolding the material for their child. However, 17% were weak at simplifying the task into manageable components, 35% were weak on sustaining the child's motivation, and 15% were weak on demonstrating how to solve the problem. It appears that roughly 15–20% of mothers, across these scaffolding variables, showed weaknesses, and these deficits might be remedied. At the same time, many mothers displayed considerable strengths, which could be tapped to foster children's learning.

The interactions themselves are fascinating. Some dyads work cooperatively, pleasantly, and purposefully on a common task. At the other end of the spectrum, some mothers are unable to explain the necessary concepts, the child is left more confused than at the beginning of the teaching time, and neither has had a satisfying interaction. Some mothers display energy and enthusiasm for the task and for mathematics, others are neutral and simply keep the nose to the grindstone, and others joke about their own lack of mathematical competence. Clearly both mother and child bring the history of their relationship to the teaching/learning task.

As expected, mother's mathematics preparation correlated significantly with the mathematical content of her teaching and with the quality of her scaffolding (considering just mothers whose child was incorrect at baseline). It is an important, if seemingly obvious, finding that mothers with more mathematics preparation are better able to help their children with difficult mathematics problems. At the same time, the correlations were not large (.23 for math composite, .27 for scaffolding composite). This suggests that other factors, in addition to the mother's mathematics preparation and knowledge, contribute to the quality of her help. These may include the quality of her interpersonal sensitivity and her ability to read non-verbal cues. It is noteworthy that mothers' math self-confidence showed a stronger correlation with the quality of the mathematical content of their teaching than mothers' mathematics preparation did. Mother's general educational level also showed strong correlations with both the quality of mathematical content and the quality of scaffolding. These findings again point to the importance of factors in addition to mothers' mathematical knowledge. These variables deserve consideration in future research.

The findings for mother's education and mathematics preparation highlight the inequities in resources that are available to students when completing homework assignments. School–family partnerships that provide resources for parents will be important in increasing the knowledge of parents who do not have a strong mathematics background.

Our results indicated that children spent on average 23 min per day on mathematics homework, with an average of 8 min of help from parents. We know of no previous study that has obtained this information. The closest equivalent is research by Chen and Stevenson (1989), who found that American children in 5th grade spent between 252 and 433 min (across 3 studies) per *week* in homework, across all school subjects. If we assume that students do mathematics homework 5 days a week, that would yield an estimate of weekly time in our study of 115 min per week, which seems within range of the values reported by Chen and Stevenson, given that theirs was a total over all subjects.

Previous research has found that mothers are the chief source of homework help, even in areas such as mathematics (Crystal & Stevenson, 1991). We obtained somewhat different results, with much more evidence of father involvement and shared helping by parents. One explanation for the discrepancy lies in the fact that we collected our data more than 10 years after Crystal and Stevenson did. American cultural beliefs in the importance of husbands sharing home responsibilities have increased over several decades, so that father involvement with children is an ideal in many families today (Hyde, Essex, & Horton, 1993; Jump & Haas, 1987; Volling & Belsky, 1991). Behaviors have changed as well. Recent data indicate that fathers in dual-earner couples spend nearly as many hours per week with their children (23.0 h) as mothers do (26.5 h), in contrast to 1981, when men in dual-earner couples spent only 17 h per week with their children (Sandberg & Hofferth, 2001).

Our findings concerning mothers' beliefs in factors influencing students' mathematics performance replicated prior findings by Stevenson et al. (1993; see also Li, 2005). Teachers and innate intelligence were the two most important factors, and studying hard and home environment were considered less important. Given these beliefs, it is not surprising that parents place little emphasis on homework (effort), and that few wish their children had more. As Stevenson and co-workers noted, these American cultural beliefs may be partly responsible for the lesser mathematics performance of American children, compared with Asian children. If American children have difficulty with a particular type of problem or score poorly on a test, they may conclude that they lack innate ability, and give up. In contrast, if effort is considered a major factor in performance, difficulties with learning are attributed to lack of effort and may be resolved by working harder, e.g., doing more homework. Parenting programs are needed to help to shift Americans' cultural beliefs toward recognizing the importance of effort, homework, and the home environment in students' mathematics learning.

To our knowledge, no previous study has asked parents about their goals in helping with mathematics homework (but see Hoover-Dempsey, Bassler, and Burow (1995), who inquired about homework broadly). The most common goals were helping the child achieve understanding and competence with mathematics problems, and helping the child to become independent in working problems, both of which are important goals. Relatively few parents stressed real-life applications and the importance of mathematics to the child's future. This last finding is troubling given Eccles' expectation x value model of choosing to take optional, challenging mathematics courses (e.g., Eccles, 1994). Students need to understand the value of mathematics to motivate them to pursue it as an achievement goal in high school and beyond. Parents may need assistance in recognizing the importance of mathematics and being aware of concrete examples of its utility.

7.1. Limitations

One limitation is that the sample is predominantly White. The findings cannot be generalized to ethnic minority populations, who may differ in patterns of interactions, mother's preparation, and other relevant variables. Future research could profitably investigate the nature of these interactions in other ethnic groups.

A second limitation is that this protocol does not tell us how mothers would perform in the most naturalistic setting, without any briefing from us. We opted for the briefing to guard against having a sizeable proportion of the interactions be completely unsuccessful because the mother went into them "cold." The briefing was not extensive. We view it as comparable to the information that a parent could gain from a brief glance at the child's textbook when helping with real homework, or from a family math program initiated by the schools in which information for parents is sent home (e.g., Whitin, 2004).

A third possible limitation is that only 5 min of mother–child interactions were analyzed. In fact, this amount of time is close to the average of 8 min of homework help that parents give per day as reported by our sample. It therefore represents a realistic length of interaction for many families. Moreover, our observations indicated that at the end of 5 min the child either understood the problems, or the pair was proceeding down a path that would not lead to mastery that day.

7.2. Conclusion and implications

Parents are a largely untapped resource for improving the mathematics performance of American children. Many mothers in this study were able teachers of pre-algebra problems. Others, however, had great difficulty and failed to discuss important concepts such as the equal sign and sides of the equation. Mothers with more education, mathematics preparation, and self-confidence performed better. This study raises many questions. For example, how might curricular materials be designed to provide better support for parents as they assist their children in learning mathematics? In regard to educational policy, if parents are to be better resources for their children's mathematics learning, we must consider school–family partnership programs in which parents are offered coaching in the kinds of problems their children are learning, and in scaffolding techniques for supporting their child's learning. The results suggest that improving mothers' math self-confidence will be at least as important as showing them how to solve a particular class of problems.

In regard to research, we need to know much more about parent–child homework interactions. One important question is how these interactions vary as a function of the child's age and the level of mathematics that is in the ZPD at that age. Anecdotally, parents report that there is a grade level or level of mathematics at which they could no longer help. Identifying this level — and it will differ for different parents — will help schools be maximally effective and efficient in deciding when they should offer support for parents. Studying these interactions in other nations, particularly those in which children perform better than American children, will also provide important information.

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