Who Benefits from Diagrams and Illustrations in Math Problems? Ability and Attitudes Matter

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Summary: How do diagrams and illustrations affect mathematical problem solving? Past research suggests that diagrams should promote correct performance. However, illustrations may provide a supportive context for problem solving, or they may distract students with seductive details. Moreover, effects may not be uniform across student subgroups. This study assessed the effects of diagrams and illustrations on undergraduates' trigonometry problem solving. We used a 2 (Diagram Presence) × 2 (Illustration Presence) within-subjects design, and our analysis considered students' mathematics ability and attitudes towards mathematics. Participants solved problems more accurately when they included diagrams. This effect was stronger for students who had more positive mathematics attitudes, especially when there was an illustration present. Illustrations were beneficial for students with high mathematics ability but detrimental for students with lower ability. Considering individual differences in ability and attitude is essential for understanding the effects of different types of visual representations on problem solving. Copyright © 2017 John Wiley & Sons, Ltd.

Mathematics textbooks often include mathematically relevant visuals, such as diagrams and graphs, as well as visuals that are less mathematically relevant, such as decorative images and photographs. Visuals are used to capture students' interest, illustrate concepts and procedures, provide context for problem-solving exercises, and for many other functions (e.g., Carney & Levin, 2002; Mayer, Sims, & Tajika, 1995). Given the large number of visual representations used in mathematics, it is important to understand how different types of visual representations affect students' learning and problem solving.

Existing research about the effects of visual representations presents a complex picture. Some studies report beneficial effects of visual representations (e.g., Hegarty & Kozhevnikov, 1999); others report detrimental effects (e.g., Berends & van Lieshout, 2009), and still others report no effects (Dewolf, van Dooren, Ev Cimen, & Verschaffel, 2014) or mixed effects (e.g., Magner, Schwonke, Aleven, Popescu, & Renkl, 2014; McNeil, Uttal, Jarvin, & Sternberg, 2009). In many studies, findings differ across subgroups of students, particularly subgroups based on student ability (e.g., Berends & van Lieshout, 2009; Booth & Koedinger, 2012).

The aim of the present study is to better understand when and how visuals support students' mathematics problem solving. Visuals with different properties likely have different effects, accounting for some of the complexity in findings to date. Specifically, diagrams may support students' mathematics problem solving by emphasizing relevant spatial information that is not easily available from text or by making critical information more salient (e.g., Davenport,

Yaron, Klahr, & Koedinger, 2008; Larkin & Simon, 1987). However, illustrations that contain no relevant mathematical information, such as an illustration of a person's face, may not support learning or problem solving at all, or may do so via students' engagement in the material.

Furthermore, characteristics of individual students, such as their level of prior knowledge (e.g., Kalyuga, Ayres, Chandler, & Sweller, 2003) or their cognitive resources (Kirschner, Paas, Kirschner, & Janssen, 2011; Moreno, 2004), may moderate the effects of visual representations on learning and problem solving. Noncognitive factors, such as interest, may moderate the effects of visuals, as well (Leutner, 2014).

In this research, we investigate the effects of two types of visual representations, diagrams and illustrations, on undergraduates' performance on trigonometry problems. We investigate how these visual representations affect students' problem solving and their evaluations of problems, and we examine whether these effects are moderated by students' mathematics abilities and their attitudes towards mathematics.

Much of the existing research on visual representations focuses on student learning (e.g., Mayer, 2008). As with learning, successful problem solving requires students to integrate verbal and visual sources of information with each other and with prior knowledge. Thus, we draw on existing research on learning to predict how and when visual representations will affect students' problem solving. To understand the mechanisms that might give rise to beneficial or negative effects of visual representations, we consider three perspectives: a *cognitive* perspective, which focuses on the nature of the information contained in visual representations and on the working memory demands of processing visual representations; a *contextualization* perspective; and a *motivation* perspective.

Positive effects of visuals

Content-relevant visual representations may support students' construction of mental representations of complex phenomena and problem-solving scenarios (e.g., Mayer,

2009). Two broad cognitive frameworks underlie much of the existing research: the cognitive theory of multimedia learning (e.g., Mayer, 2009; Mayer & Moreno, 2003) and cognitive load theory (e.g., Chandler & Sweller, 1991; Paas, Renkl, & Sweller, 2003; Plass, Moreno, & Brünken, 2010; Sweller, 2004). These theories focus on how information is processed. Although the theories have some important differences, a central idea of both theories is that the structure of the cognitive system imposes limits that influence how learners select, organize, and integrate information.

The *multimedia principle*, derived from the cognitive theory of multimedia learning, holds that words and relevant visuals lead to greater learning than words alone (e.g., Butcher, 2006; Mayer, 2009; Mayer & Anderson, 1992; Sung & Mayer, 2012; see review by Carney & Levin, 2002). This principle is based on the assumption that separate visual and verbal channels take in and process information before the two sources are integrated. Having information in both channels leads to more complete and flexible mental models.

However, when information is presented in both visual and verbal channels, students must engage in substantial processing to select and integrate the information from both sources and their prior knowledge (Mayer, 2009), and thus, other factors may moderate the principle's effects. Some of these moderating factors include learners' processing resources, prior knowledge or ability, the relationship between the verbal and visual information, and the difficulty of the problem (e.g., Kalyuga, Chandler, & Sweller, 1998, 1999; Kühl, Scheiter, Gerjets, & Gemballa, 2011; Sung & Mayer, 2012; van Merriënboer & Sweller, 2005).

The contextualization perspective holds that contextualizing or 'grounding' math problems in real-world scenarios supports successful problem solving (Belenky & Schalk, 2014; Goldstone & Son, 2005; Koedinger & Nathan, 2004). Contextualization can be achieved in various ways, such as by presenting problems in familiar contexts (e.g., Koedinger, Alibali, & Nathan, 2008) or by including visual details in visual representations (e.g., photographs; Belenky & Schalk, 2014). Contextualization with realistic or familiar content is thought to promote problem solving because it supports individuals in understanding problems and in integrating prior knowledge with the problems. Across a variety of mathematical domains, researchers have found that contextualizing mathematics problems can foster performance and learning (e.g., Carraher, Schliemann, Brizuela, & Earnest, 2006; Koedinger et al., 2008; Wilhelm & Confrey, 2003).

Mathematics problems are often contextualized using text, such as in story problems, but they can also be contextualized with visual representations (e.g., McNeil et al., 2009). However, there is little data about how visual representations such as context-relevant illustrations affect performance. It again appears that individual difference factors such as ability, attitudes towards mathematics, and problem difficulty (Belenky & Schalk, 2014; Cooper & Walkington, 2013; Koedinger et al., 2008; Magner et al., 2014; Walkington, 2013) may moderate the effects of contextualization; however, there are mixed findings about which combinations are most beneficial.

Finally, visual representations may affect learning by increasing students' motivation to attend to materials or persist in the face of difficulty (Moreno, 2005, 2006). For example, in one study, participants who viewed perceptually rich visual representations reported higher motivation and found the learning task to be less difficult than those who viewed bland representations (Plass, Heidig, Hayward, Homer, & Um, 2014). These authors suggested that the visual detail fostered positive emotions in learners, increasing their motivation and persistence, resulting in better comprehension of the material. Similarly, Belenky and Schalk (2014) argue that the extent to which a representation is grounded can influence interest, with purely diagrammatic representations being associated with lower interest than visual representations with more detail. However, it is worth noting that not all studies reveal beneficial effects of visual representations on problem evaluations or other motivational outcomes (Harp & Mayer, 1997; Lehman, Schraw, McCrudden, & Hartley, 2007; Mayer & Estrella, 2014).

Negative effects of visuals

Although contextual and motivational perspectives suggest that nonessential details of visual representations may have positive effects on students' learning and problem solving, extraneous features can also be detrimental to learning, particularly when they are alluring or distracting (e.g., Garner, Gillingham, & White, 1989; Harp & Mayer, 1997, 1998; Mayer, 2009; for a review, see Rey, 2012). The *coherence principle*, derived from the cognitive theory of multimedia learning, holds that extraneous features of the material are detrimental to learning, because adding interesting but irrelevant material can overload the visual or verbal pathways (Mayer, 2009). The effort devoted to cognitive activities that are not related to learning limits the cognitive resources available for learning (Sweller, 2005; Sweller, Van Merriënboer, & Paas, 1998).

This phenomenon, often called the *seductive details effect*, applies to alluring details in text or illustrations (Harp & Mayer, 1997, 1998). In one study, Harp and Mayer (1997) added such irrelevant, but attention-catching, seductive details to passages about the process of lightning. The extraneous details included information about lightning hitting swimmers or airplanes—topics of interest to readers, but irrelevant to the scientific process that was the target for learning. Seductive details in text, illustrations, or both reduced participants' recall of target information and reduced their application of the material to new problems. A few other studies have yielded mixed or null effects of seductive details (e.g., Chang & Choi, 2014; Schraw, 1998; Schraw & Lehman, 2001); however, the majority of findings in the literature indicate that seductive details are negative. Indeed, a recent meta-analysis (Rey, 2012) of 39 experimental effects revealed a significant negative effect of seductive details, both for memory retention and for transfer. Moreover, this pattern held for seductive details in both text and illustrations.

Of course, not all details are 'seductive' in the sense of being extraneous to the content at hand. However, even perceptual details that are content-relevant do not always

support successful learning and problem solving. For example, Butcher (2006) found that although diagrams generally supported learning about the circulatory system, simplified diagrams were more beneficial than detailed diagrams. Similarly, McNeil et al. (2009) found that when children solved word problems about money, perceptually sparse money representations were more beneficial than perceptually rich ones.

Individual differences

Both the positive and negative effects of visual representations on students' learning and problem solving may depend on characteristics of the students, such as their level of prior knowledge and their attitudes towards the task. Indeed, the predictions made by the cognitive theory of multimedia learning and by cognitive load theory hinge on learners' capacity for processing information and whether or not the available information would overload their capacity, both of which should vary across learners.

Many studies have revealed effects of ability or prior knowledge on students' integration of text and visuals (e.g., Kalyuga, 2007) and on students' susceptibility to seductive details (e.g., Park, Moreno, Seufert, & Brünken, 2011). For example, Kalyuga et al. (1998) found that learners with less prior knowledge benefited from diagrams that were more detailed, whereas learners with more prior knowledge benefited from sparser diagrams. Although learners with less prior knowledge required the extra detail to make use of diagrams during learning, those with greater expertise reported lower cognitive load when using sparse diagrams. Thus, the authors argued that the diagrams with detailed text resulted in unnecessary additional cognitive load, dampening learning.

Magner et al. (2014) found that learners with low prior knowledge were more susceptible to the detrimental effect of mathematically irrelevant, but contextualizing details added to a sparse diagram. Decorative details added to geometry diagrams decreased learning for eighth-grade students with low prior knowledge, but fostered learning for students with very high prior knowledge. Among students with low prior knowledge, learning geometry may require substantial cognitive resources. Having to process seductive details along with the sheer demands of integrating text and visuals may exceed their available working memory, leading to detrimental effects. In contrast, Magner et al. (2014) suggest that students with very high prior knowledge may rely on prior knowledge to reduce the overall demands of the learning task on their working memory resources. In this case, the learners' processing of irrelevant details did not exceed their capacity, and the illustrations enhanced students' interest, thus having a net positive effect on learning. Taken together, these studies show that whether details added to sparse diagrams support or detract from learning depends on both the specifics of those details and the learners' prior knowledge levels. These studies also underscore the importance of considering the effects of illustrations on students' interest in and evaluations of the materials.

Students' attitudes towards the tasks or the domain may also affect how students process visuals. Visuals may increase interest and motivation in a general way, as discussed earlier, but students' interest and attitudes may also moderate the effects of visuals. In general, students' mathematics attitudes are positively related to their mathematics achievement (e.g., Ma & Kishor, 1997; Singh, Granville, & Dika, 2002). However, there are many dimensions of students' attitudes towards math, including their own perceived ability, their interest in mathematics, and their perceptions of the value of mathematics, all of which have positive correlations with school achievement (Eccles & Wigfield, 1995; Sirmaci, 2010). These affective and motivational factors may influence students' engagement with the materials and thereby affect learning (Moreno, 2005; Moreno, 2006; Park, Flowerday, & Brünken, 2015). However, little research has directly tested these predictions about how affective and motivational factors affect students' mathematics problem solving.

A few experimental studies have directly investigated the moderating effects of undergraduates' attitudes towards math, but findings are mixed. Ainley, Hidi, and Berndorff (2002) found that when learning from text, students with greater interest in the task spend more time engaging in learning than those with less interest. Hattikudur, Sidney, and Alibali (2016) found that one instructional approach (supporting procedure comparison) mattered only for students who reported disliking math, who were conceivably less likely to expend the cognitive effort needed to learn in the less supported instructional condition. Durik and Harackiewicz (2007) found that decorative details added to visual representations were beneficial for students who were not interested in learning mathematics but detrimental for those who were already interested in learning. Conversely, Wang and Adesope (2016) found that decorative details had negative effects among students with lower interest in the learning topic but positive effects among students with higher interest.

In sum, students who have positive attitudes towards a task tend to employ more cognitive resources during that task, thus changing the nature of their engagement with the materials. These findings also highlight that the available cognitive resources are different from the resources that a student *chooses* to apply to a task. Thus, it seems likely that students' attitudes may moderate the potentially beneficial or detrimental effects of visual representations (e.g., Durik & Harackiewicz, 2007; Wang & Adesope, 2016).

Importantly, this body of prior research has focused primarily on students' *interest* in mathematics, despite the multi-faceted nature of students' attitudes towards mathematics (e.g., Eccles & Wigfield, 1995; Walkington, Cooper, & Howell, 2013). Although dimensions of students' attitudes are often correlated (e.g., Chouinard, Karsenti, & Roy, 2007; Eccles & Wigfield, 1995), it seems possible that students' interest in mathematics and their attitudes about its value may have differential effects on their processing of diagrams and illustrations. For example, both Durik and Harackiewicz (2007) and Wang and Adesope (2016) found that students' interest moderated the effect of decorative details—a feature of illustrations rather than sparse diagrams.

Students' attitudes about the value of mathematics may also influence how they process visual representations, but there is little research on this issue. In general, students with

higher value attitudes are more highly motivated to master a task, and in turn, they expend more effort in the face of difficulty (Chouinard et al., 2007). This suggests that students with higher value attitudes may be more motivated to take advantage of the spatial information presented in visual representations, leading to greater effects of diagrams and illustrations. Little research has directly examined the effects of students' interest and value attitudes on their processing of diagrams, illustrations, or their combination. Thus, one goal of the current study is to explore the relationship between students' mathematics attitudes and their use of visual representations in greater depth.

Current study

In this study, we focus on two kinds of visual representations that are ubiquitous in mathematics textbooks: simplified diagrams and contextually relevant, perceptually rich illustrations. Combined, these representations yield perceptually rich diagrams with illustrative features. Such visual representations contain both relevant and irrelevant information. The irrelevant information could be problematic if it overloads available resources, but it could be beneficial through its effects on engagement. In this study, we investigated how these diagrammatic and illustrative visual representations affect problem solving among students of varying ability levels and with varying attitudes towards mathematics. We chose to investigate these questions in trigonometry because it is a highly spatial domain, in which visual representations may be particularly valuable. We used problems for which multiple solution paths were possible, so that processing capacity would be taxed as solvers search for an appropriate solution path (Sweller, 1988).

We presented undergraduate students with trigonometry problems in a 2 (Diagram Presence: yes or no) × 2 (Illustration Presence: yes or no) within-subjects design. The problems were contextually grounded via the story problem text in all conditions. *Diagrams* presented key information from the problems in schematic, spatial form. The illustrations, which we refer to as *contextual illustrations*, included graphics that provided mathematically relevant information in the spatial layout. They also contained decorative features that were not relevant to solving the problems, but that served to further contextualize the problems.

In investigating the effects of the visuals, we consider both students' problem solving and their evaluations of the problems. We asked students to rate the difficulty and clarity of each problem, as well as to indicate their willingness to do more problems like the one under consideration. Similar questions have been used in previous studies (e.g., Kalyuga et al., 1999; Leppink, Paas, van der Vleuten, Van Gog, & van Merriënboer, 2013). We consider whether visual representations affected students' evaluations of the problems and how students' evaluations aligned with their performance.

Hypotheses

Based on the multimedia principle (e.g., Mayer, 2009) and the beneficial effects of sparse diagrams (e.g., Butcher, 2006), we hypothesized that students would be more accurate on problems with diagrams than problems presented in text only. Furthermore, based on the contextualization perspective, we hypothesized that students would be more accurate on problems with contextual illustrations than problems presented in text only.

However, we expected that combining diagrams and illustrations would hinder problem solving. According to the coherence principle, the additional detail in the illustrations should make them detrimental to students' performance, relative to diagrams alone, because that additional detail creates extraneous cognitive load. Indeed, combining the multimedia and coherence principles would imply that diagrams alone should be most beneficial, because they offer the benefits of visual representations, without the harm of seductive details. On the other hand, the benefits of diagrams alone could be enhanced by including illustrations if students' engagement were increased by the illustrations.

We anticipated that these effects would be moderated by individual differences. Based on the prior research (e.g., Kalyuga et al., 1998; Magner et al., 2014), we expected that students with lower mathematics ability would benefit less from the visual representations, particularly from contextual illustrations, than those with higher mathematics ability. Following Magner et al. (2014), we hypothesized that students with lower mathematics ability might be especially susceptible to the cognitive load imposed by the extraneous details. They might also have more difficulty discerning the critical mathematical information in the face of seductive details; therefore, they might benefit less from illustrations than students with higher mathematics ability. In contrast, students with higher mathematics ability may be able to process the trigonometric relationships more easily by relying on their prior knowledge, and the additional processing of extraneous details may not overload their working memory resources.

We also examined whether the effects of the visual representations depended on students' attitudes towards mathematics. As little research has directly addressed the role of students' mathematical attitudes in learning or problem solving with visual representations, these analyses were largely exploratory. However, based on prior studies of attitudes and mathematical learning (e.g., Ainley et al., 2002; Hattikudur et al., 2016), we expected that students with more *interest* in mathematics would devote greater cognitive resources to problem solving, dampening the negative effects of adding illustrative features to diagrams. Furthermore, we expected that students who *valued* mathematics more highly would also be more motivated to use visual representations productively. Given our prediction that diagrams would be more effective at supporting problem solving than illustrations, we expected that students' value attitudes would moderate the effect of diagrams more so than that of illustrations.

Finally, we considered whether visual representations would affect students' evaluations of the problems. We expected that students might demonstrate increased interest in problems that included illustrative details. We further expected that students' problem evaluations would align with their problem-solving success. That is, we expected that students would rate problems more positively when they had greater success in solving them.

METHOD

Participants

Participants were 92 undergraduates (*M* = 19.0 years, *SD* = 1.26, 51% female), who received credit in introductory psychology for their participation. Participants' self-reported race/ethnicity was distributed as follows: 53% White or Caucasian, 35% Asian, 3% Hispanic, 1% Black/African American, and 6% other/not specified.

Design and materials

Each participant received four problems based on a 2 (Diagram Presence) × 2 (Illustration Presence) within-subjects design, yielding four conditions: text alone, diagram alone, illustration alone, and diagram with illustrative features (Figure 1). The text of the story problem was identical in all conditions, and the text alone was sufficient to answer the problems. Condition order was counterbalanced across participants.

Each problem involved a different cover story (Appendix A). All required applying trigonometric relations to overlapping right triangles to solve for an unknown dimension, but each required different solution processes. We chose these types of problems because they lent themselves well to concrete situations and because they were at an appropriate difficulty level for undergraduate participants. The order of the cover stories was held constant. Each problem was presented on its own page, with ample space for written work. There were blank pages between problems so that participants could not see subsequent problems.

The illustrations were associated with the problem content, thus providing context. As shown in Figure 1, the illustrations were primarily decorative and also included mathematically relevant information, in that they indicated the spatial layout of the components of the problems.

Participants' math attitudes were assessed using a 12-item scale that had been used in a previous study of interest and visual representations in problem solving (Walkington et al., 2013). The scale included seven 5-point Likert items and five 3-point Likert-type items tapping self-perceived ability (e.g., 'How good at math are you?'), value (e.g., 'Math is important in everyday life.'), and interest (e.g., 'How much do you like math?'; see Appendix B for a list of items as well as the item–item correlations). To obtain a more objective measure of participants' math ability, students were also asked to report their ACT or SAT mathematics scores. For analysis purposes, ACT and SAT math scores were converted to percentiles based on The College Board's (2010) and ACT's (2011) percentile ranks for college bound seniors.

Procedure

The procedures followed were in accordance with the ethical standards of the corresponding author's institutional review board. Participants received a reference handout (with text and equations, but no diagrams) of information about triangles and trigonometric formulas along with a calculator with trigonometric functions. Participants were told that not all of the information on the handout would be needed. Participants worked through the four problems at their own pace.

After completing the problems, participants rated how difficult each problem was, how clear it was, and how willing they would be to do more problems like it, using 5-point Likert scales. While making these ratings, participants were
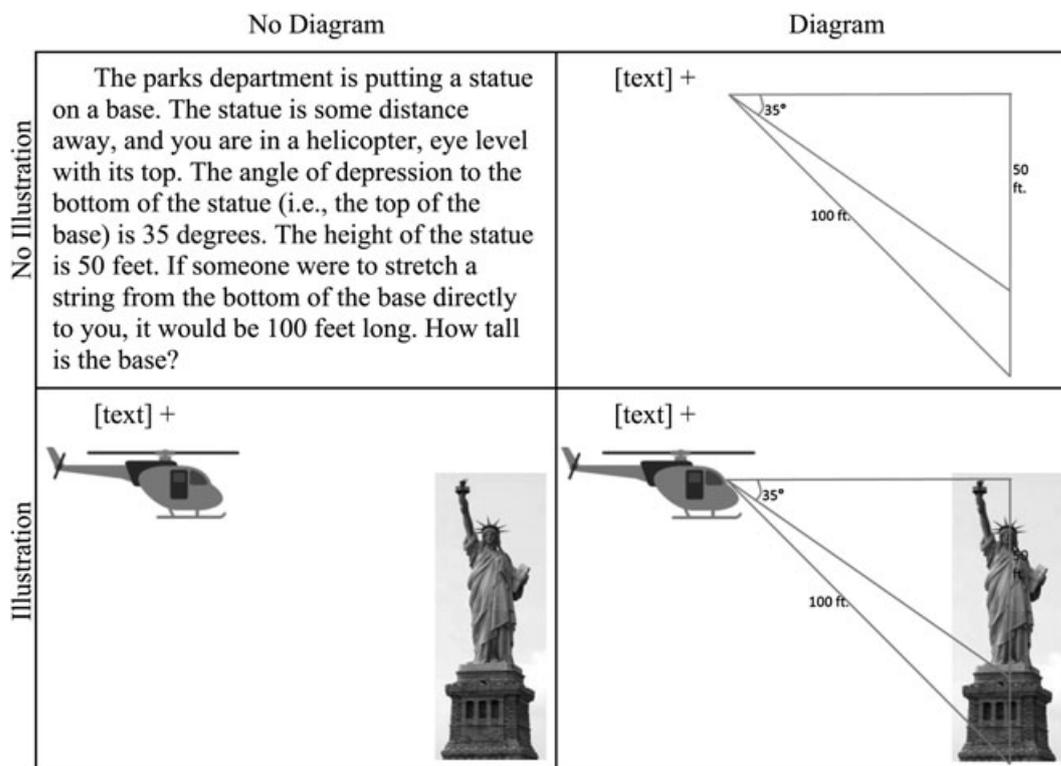

Figure 1. First cover story, shown for each visual condition

allowed to look back over the problems but not to change their answers. For analysis, ratings of problem difficulty were reverse scored, so that the measure reflects problem 'easiness'; higher ratings reflect more favorable evaluations on each measure.

Finally, participants completed background questionnaires including the math attitude scale, a math background survey, and a demographic survey.

Factor analysis

Previous research on students' attitudes towards math has identified separable sub-constructs, including interest in mathematics, perceived utility of mathematics, perceived importance of mathematics, expectancies about mathematical abilities, perceived difficulty of a mathematics task, and effort required for a mathematics task (Eccles & Wigfield, 1995). Thus, in order to examine specific relationships between each facet of participants' math attitudes and the experimental conditions, we conducted a factor analysis on the math attitude scale. About 1% of the attitude and background variable observations were missing, primarily ACT or SAT math scores. These missing data were imputed using expectation maximization estimation in SPSS based on all background variables.

Factor analysis (using principal components extraction with direct oblimin rotation, eigenvalues >1) on the math attitude scale indicated three factors were appropriate. The factor scores were obtained after removing two variables due to low communalities and/or determinants of 0 when included. These factor scores were used in the subsequent analyses. The items included in each factor are listed in Appendix B, Table B1.

The first factor incorporated five items from the scale (e.g., 'How good are you at math?') and appeared to probe students' *self-perceived mathematics ability*. Self-perceived ability was strongly correlated with ability as measured by students' ACT/SAT math percentile score, $r = .67$, $p < .01$ (Table 1). Thus, in our analysis, we relied on the more objective measure. In this regard, it is important to note that self-reported standardized test scores are highly correlated with actual scores, although they do suffer from some reporting biases (Cole & Gonyea, 2010). The second factor incorporated two items from the scale (e.g., 'I find many math problems interesting') and appeared to probe students' *interest* in mathematics. The third factor incorporated three items from the scale (e.g., 'Math is important in everyday life') and appeared to probe students' *values* for

mathematics. This factor structure is consistent with previous findings using this math attitudes scale (Walkington et al., 2013) as well as prior research on mathematics attitudes (Eccles & Wigfield, 1995).

Correlations among the factor scores and the correlations between each factor score and objective ability as measured by ACT/SAT math percentile scores are reported in Table B2. Mathematics ability, measured by students ACT/SAT math percentile scores, was significantly correlated with value and interest factor scores. However, *interest* and *value* attitudes were uncorrelated. To get a better sense of the relationships between these three variables, we examined correlations between ability, interest attitudes, and value attitudes at high and low ability by performing a median split on ability. In both the higher-ability and lower-ability groups, interest and value attitudes were uncorrelated ($r = .03$, $p = .83$, for higher-ability students and $r = .22$, $p = .17$, for lower-ability students). Similarly, ability and value attitudes were uncorrelated in both groups ($r = .19$, $p = .22$, for higher-ability students and $r = .23$, $p = .14$, for lower-ability students). However, relations between ability and interest differed for the higher-ability and lower-ability groups. In the higher-ability group, ability was marginally *negatively* correlated with interest, $r = -.30$, $p = .05$, such that students with higher ability were less likely to like math, in this range. In contrast, in the lower-ability group, ability was *positively* correlated with students' interest, $r = -.42$, $p = .01$, such that students in this range with relatively higher ability were more likely to like math.

RESULTS

Preliminary analyses

Overall, the problems given in this study were fairly challenging. Across conditions, participants answered only 50% of problems correctly, $M = 0.50$, $SD = 0.50$. Table 2 presents the proportion of participants solving the problem correctly in each condition. On average, participants had the greatest success on problems presented with diagrams and the least success on problems presented in text only. In the following sections, we present our analyses of accuracy and problem ratings as a function of condition, math ability, math interest, and math value attitudes.

Accuracy

Model specification
We analyzed accuracy on each problem using mixed effects logistic regression in the *lme4* R package (Bates, Maechler, Bolker, & Walker, 2014). The regression model included

Table 1. Correlations among factor scores and with objective ability

Measure	Ability (ACT/SAT)	Ability (factor)	Interest (factor)	Value (factor)
Ability (ACT or SAT)	—			
Ability (factor)	**.67**	—		
Interest (factor)	**.39**	.19	—	
Value (factor)	**.22**	**.51**	.12	—

Note: Bolded correlations are significant at $p < .05$.

Table 2. Proportion of participants who solved the problem correctly in each experimental condition

Condition	M	SD
Text only	0.36	0.48
Diagram only	0.59	0.50
Illustration only	0.41	0.50
Diagram and illustration	0.63	0.49

the fixed effects of diagram presence, illustration presence, math ability, math interest, and math value, with all variables mean centered. As we were interested in how individual difference variables moderated the effects of diagram and illustration presence, we included the three-way interactions between diagram, illustration, and each individual difference factor and their lower order two-way interactions. We were unable to fit the maximal random effects structure due to model convergence failure. As a first step towards simplifying the model, we eliminated higher-order random slopes until the model converged. Then, in order to determine the most parsimonious random effects structure while avoiding overparameterization, we followed the procedure outlined by Bates and colleagues for simplification (see Bates, Kliegl, Vasishth, & Baayen, 2015, for a full discussion). We ran a principal component analysis of the random effects variance–covariate estimates and eliminated random effects accounting for the least amount of variance until the model was no longer overparameterized. The resulting model included only the by-subject random intercept and the by-item random intercept.

To evaluate significance of model terms, we report the chi-squared statistic from an analysis of variance comparison of the log likelihoods of nested models with and without the term in question. To examine the nature of the interaction effects, simple effects of diagram and illustration were calculated at ± 1 *SD* of the continuous variables. Full model results are reported in Table 3.

Model results

As predicted, diagrams promoted greater accuracy, $OR = 5.94$, $\chi^2(1) = 27.53$, $p < .001$. However, the strength of this effect was moderated by how much participants valued math, $OR = 2.81$, $\chi^2(1) = 8.05$, $p < .01$ (Figure 2). Diagrams had an increasingly beneficial effect for participants with more positive attitudes about the value of math. Among those who valued math more highly (1 *SD* above the mean), diagram presence had a large positive effect, $OR = 15.86$, $p < .01$.
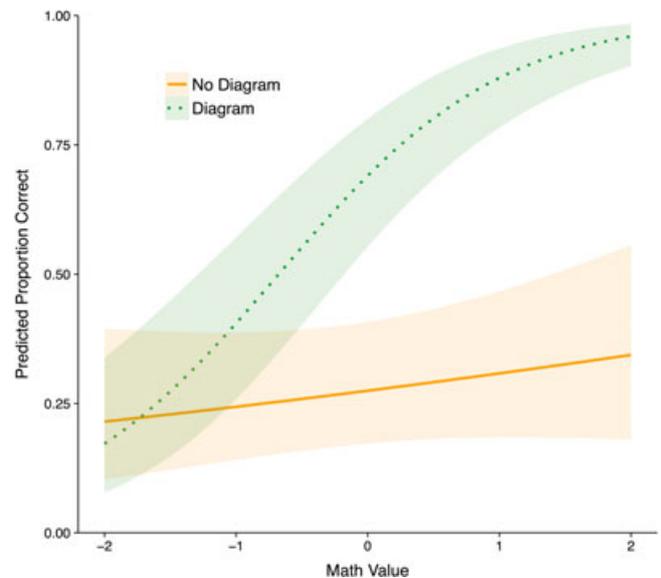

Figure 2. The beneficial effect of diagrams was larger among participants with more positive attitudes about the value of math. Lines show predicted proportion correct across the range of value attitude levels for diagram (dotted line) and no diagram (solid line) conditions. Error bands represent ± 1 *SE*; predicted values and errors were calculated using the R package *AICcmodavg* (Mazerolle, 2013). [Colour figure can be viewed at wileyonlinelibrary.com]

However, among those who valued math less highly (1 *SD* below the mean), diagrams had no effect, $OR = 2.80$, $p = .15$.

In contrast, illustrations did not have an overall beneficial effect on accuracy, $\chi^2(1) = 0.02$, $p = .89$. However, there was a significant interaction of illustration and ability, $\chi^2(1) = 15.21$, $p < .001$. Illustrations were beneficial for participants with high mathematics ability (1 *SD* above the mean, Figure 3); for these participants, the odds of solving a problem correctly were 6.39 times higher when an illustration was present than when it was absent, $p < .001$. In contrast, illustrations were detrimental for participants with lower mathematics ability (1 *SD* below the mean); for these participants, the odds of solving a problem correctly were

Table 3. Results of the mixed effects logistic regression model of participant performance

Terms	*b*	95% CI	*SE*	OR	χ^2	*p*
Diagram Presence	1.78	1.03, 2.54	0.39	5.94	27.53	<.001*
Illustration Presence	0.05	−0.64, 0.73	0.35	1.05	0.02	.89
Math Value	0.67	0.12, 1.21	0.28	1.95	34.71	<.001*
Math Ability	0.11	0.05, 0.18	0.03	1.12	17.01	<.001*
Math Interest	0.23	−0.28, 0.74	0.26	1.26	0.84	.36
Diagram × Illustration	0.29	−1.06, 1.63	0.69	1.33	0.16	.69
Diagram × Math Value	1.03	0.29, 1.77	0.38	2.81	8.05	<.01*
Diagram × Math Ability	−0.08	−0.16, 0.008	0.04	0.92	3.41	.06
Diagram × Math Interest	0.89	0.18, 1.59	0.36	2.43	6.83	<.01*
Illustration × Math Value	−0.07	−0.78, 0.65	0.36	0.94	0.07	.80
Illustration × Math Ability	0.16	0.07, 0.25	0.05	1.17	15.21	<.001*
Illustration × Math Interest	0.23	−0.44, 0.90	0.34	1.26	0.46	.50
Diagram × Illustration × Value	0.03	−1.38, 1.43	0.72	1.03	0.03	.87
Diagram × Illustration × Ability	−0.07	−0.23, 0.10	0.09	0.94	0.62	.43
Diagram × Illustration × Interest	1.79	0.41, 3.15	0.70	5.96	7.18	<.01*

Note: Parameter estimates are given both in the original log odds (*b*) and converted to ORs. Standard errors are standard errors of the log odds (*b*). CIs are Wald approximations. Given *p*-values are associated with the χ^2 statistics from each analysis of variance comparison of the log likelihoods of two nested models. Asterisks indicate significance at $\alpha = .05$. Significant terms are also listed in bold text. CI, confidence interval; OR, odds ratio.
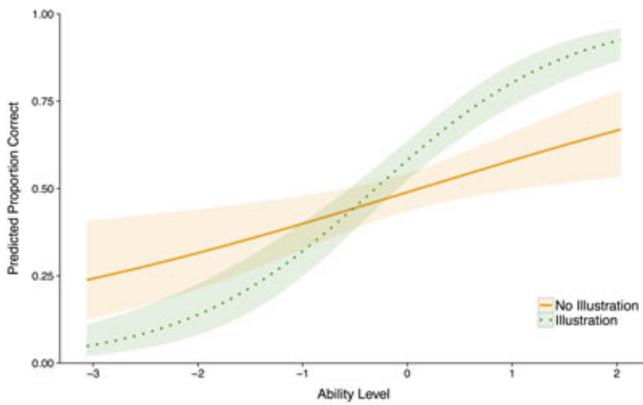
Figure 3. Illustrations helped performance for individuals with higher mathematics ability but were detrimental for individuals with lower mathematics ability. Lines show predicted proportion correct across the range of ability levels for illustration (dotted line) and no illustration (solid line) conditions. Error bands represent ±1 *SE*; predicted values and errors were calculated using the R package *AICcmodavg* (Mazerolle, 2013). [Colour figure can be viewed at wileyonlinelibrary.com]

a positive effect when illustration was also present, OR = 36.15, $p < .001$.

Problem favorability ratings

We first examined the correlations among the three problem rating measures. Each pair of measures was highly correlated: problem easiness and their willingness to do more similar problems, $r = .33$, $p < .01$, problem easiness and clarity, $r = .55$, $p < .01$, and problem clarity and willingness, $r = .48$, $p < .01$. For simplicity of reporting, we averaged these measures together into a single rating of problem favorability. The conclusions reported here are the same for each measure if analyzed separately, as well.

Model specification
As with accuracy, we analyzed problem favorability ratings using mixed effects linear regression in the *lme4* R package (Bates et al., 2014). As for accuracy the model included the fixed effects of diagram presence, illustration presence, math ability, math interest, and math value, as well as the three-way interactions between diagram, illustration, and each individual difference factor and their lower-order two-way interactions. We simplified the random effects structure as described earlier, with the resulting model including only the by-subject random intercept and the by-item random intercept. To evaluate significance, we report Type III Wald *F*-tests with Kenward–Roger degrees of freedom. Full model results are reported in Table 4.

Model results
Participants rated problems with a diagram more favorably than those without a diagram, $b = .26$, $F(1, 86.40) = 12.58$, $p < .001$, and problems with an illustration more favorably than those without an illustration, $b = .29$, $F(1, 86.30) = 13.23$, $p < .001$. In addition, participants who valued math more highly, $b = .32$, $F(1, 84.49) = 29.26$, $p < .001$, and those with higher ability, $b = .02$, $F(1,$

0.16 times lower when an illustration was present than when it was absent, $p = .01$.

Finally, there was a three-way interaction between diagram presence, illustration presence, and participants' interest in mathematics, OR = 5.96, $\chi^2(1) = 7.18$, $p < .01$. When no illustration was present, there was a positive effect of diagram presence across the range of participant interest, OR = 5.20, $p < .001$, and this effect was not moderated by participant interest, OR = 1.00, $p = .99$ (Figure 4(a)). However, when an illustration was present, the effect of diagram was moderated by participant interest, OR = 5.26, $p < .01$ (Figure 4(b)). Among participants with low interest in mathematics (1 *SD* below the mean), diagram presence was not beneficial when the illustration was also present, OR = 1.11, $p = .88$. In contrast, among participants with high interest in mathematics (1 *SD* above the mean), diagram had
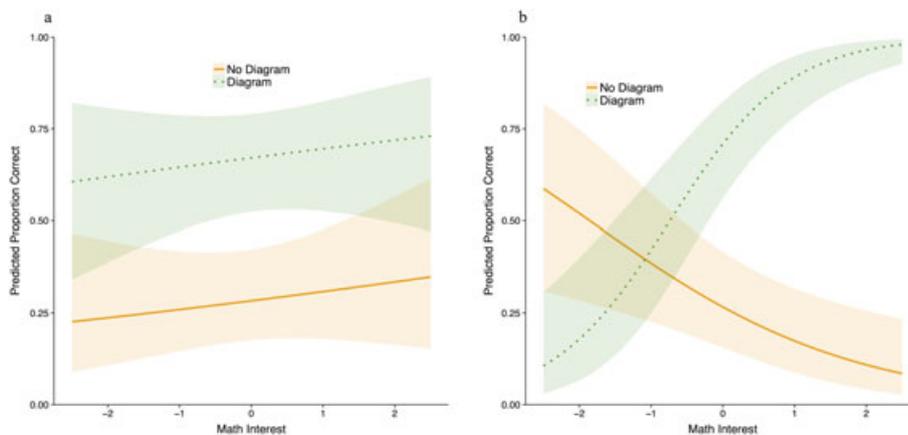

Figure 4. When no illustration was present, diagrams were beneficial across the range of participant interest (a). When an illustration was present, the effect of diagram was greater among individuals with higher interest in math (b). Lines show predicted proportion correct across the range of math interest levels for diagram (dotted lines) and no diagram (solid lines) conditions. (a) Shows the effect of diagram when no illustration was present. (b) Shows the interaction of diagram and interest when an illustration was present (b). Error bands represent ±1 *SE*; predicted values and errors are calculated using the R package *AICcmodavg* (Mazerolle, 2013). [Colour figure can be viewed at wileyonlinelibrary.com]

Table 4. Results of the mixed effects linear regression model of problem favorability ratings

Terms	b	SE	95% CI	F	df res	p
Diagram Presence	0.26	0.07	0.13, 0.40	12.37	250.13	<.001*
Illustration Presence	0.29	0.07	0.14, 0.42	15.45	251.30	<.001*
Math Value	0.32	0.06	0.20, 0.45	28.47	84.50	<.001*
Math Ability	0.02	0.01	0.01, 0.03	21.18	84.72	<.001*
Math Interest	0.05	0.06	−0.07, 0.16	0.64	85.00	.43
Diagram × Illustration	0.11	0.15	−0.17, 0.42	0.56	250.22	.46
Diagram × Math Value	0.06	0.08	−0.08, 0.22	0.55	250.96	.46
Diagram × Math Ability	−0.001	0.01	−0.02, 0.01	0.02	250.21	.89
Diagram × Math Interest	0.10	0.08	−0.06, 0.25	1.82	250.75	.18
Illustration × Math Value	−0.06	0.08	−0.22, 0.10	0.50	250.83	.48
Illustration × Math Ability	−0.005	0.01	−0.02, 0.01	0.47	250.86	.49
Illustration × Math Interest	0.05	0.08	−0.04, 0.24	1.46	252.55	.23
Diagram × Illustration × Value	−0.06	0.16	−0.34, 0.25	0.12	250.34	.72
Diagram × Illustration × Ability	−0.004	0.01	−0.03, 0.02	0.09	250.35	.76
Diagram × Illustration × Interest	0.11	0.15	−0.17, 0.42	0.53	250.15	.47

Note: Standard errors are standard errors of the parameter estimates (*b*). CIs are calculated from a parametric bootstrapping procedure. Here, we report Type III Wald *F*-tests with Kenward–Roger residual degrees of freedom and their associated *p*-values. Asterisks indicate significance at α = .05. Significant terms are also listed in bold text. CI, confidence interval.

84.68) = 20.75, *p* < .001, rated problems more favorably than those who valued math less highly and those with lower ability, respectively. None of the interactions were significant.

DISCUSSION

As expected, on average, participants were more likely to accurately solve problems with diagrams than problems without diagrams, and this effect was more pronounced for students with more positive attitudes towards mathematics. Illustrations, in contrast, were not beneficial across the board. Students with higher mathematics ability were more likely to accurately solve problems with illustrations than problems without illustrations. However, the reverse was true for students with lower ability, who were *less* likely to accurately solve problems with illustrations than problems without illustrations. Furthermore, when diagrams were present along with illustrations, students with low interest in math no longer benefited from the diagrams. Here, we consider how these results align with the predictions derived from the different theoretical frameworks.

Alignment of accuracy findings with theoretical positions

According to the *multimedia principle*, problems with visual representations should be solved more successfully than problems presented as text only. This held true for diagrams, and for illustrations, it held true for a subset of participants, namely, those with high ability. Overall, there was fairly strong support for the multimedia principle.

The effects of illustrations can also be considered with respect to the *coherence principle* and the *contextualization perspective*, which make opposite predictions. The contextualization perspective predicts that contextual illustrations should be beneficial because additional detail helps students ground the problems and connect them to prior knowledge. Indeed, the beneficial effect of illustrations for students with high ability suggests that this mechanism might have been operating. The coherence principle, in contrast, predicts that contextual illustrations should be harmful, because they contain extraneous details (Mayer, 2003). This was the case for students with low ability in this study; for these students, the negative effects of extraneous details overrode any potentially beneficial effects of contextualization. Both mechanisms could operate at the same time; for students with median levels of ability, the potential benefits of contextualization could cancel out the harmful effects of lack of coherence.

Note that the coherence principle addresses the exclusion of *extraneous* details, and not all details are extraneous (Belenky & Schalk, 2014). In this study, the contextual illustrations depicted the spatial relations in the problems as well as the objects in the problems, so some aspects of the illustrations (i.e., the spatial relations) were not extraneous, whereas other aspects (i.e., the perceptual details of the depicted objects) were extraneous. Indeed, the relevant spatial information in the illustrations may have mitigated the potential disruption of the other seductive details for students with high ability. In light of these findings, and in line with others (e.g., Mayer & Estrella, 2014), we suggest that researchers need more nuanced ways to determine what aspects of an illustration are truly extraneous.

In the same spirit of developing more nuanced principles, we suggest that the effect of seductive details may also depend on the total amount of effort a learner must expend on a task (Moreno & Park, 2010; Park et al., 2011). Thus, one way to think about the differing effects of contextual illustrations for students with different ability levels is in terms of how much effort students require to solve the problem. Students with lower mathematics ability may need to put forth more effort than students with higher ability to encode features of the problems within the contextual illustrations, as they may have less relevant prior knowledge to draw on to guide their encoding. Thus, for low-ability students, extraneous information may overload the cognitive system, overriding any potential benefits of contextualization. For such students, 'less is more' (Mayer, 2014). Individuals with higher ability, in contrast, may be better able to draw on relevant prior knowledge to guide their encoding of key information (e.g., trigonometric relationships) from within

the illustrations. For such students, their prior mathematical knowledge supports their efficient problem solving.

It is worth noting that some research has shown greater benefits of contextual information for students with lower math ability on harder problems (Walkington, 2013). However, in the present study, contextual information in illustrations was negative for such students. One possibility is that the specific contextual information we used may not have connected well to students' prior experiences, and this may have been especially problematic for lower-ability students. Although the problems used in the study are representative of the types of problems commonly found in mathematics textbooks, they were not authentic problems of the sort that students frequently encounter in their everyday lives (e.g., it is unlikely that anyone would ever drop a string from a helicopter as in the *statue* problem). It may be the case that contextualization is most helpful when students have *relevant* prior experiences. Future research is needed to explore the specific mechanisms by which contextualization affects problem solving.

We also found that the effects of visual representations depended on students' attitudes towards mathematics in two distinct ways. First, the beneficial effects of diagrams were not observed for students with less interest in math, when an illustration was also present. It is possible that students with less interest in mathematics were not as engaged in the materials; thus, they expended less cognitive effort on problem solving than their more interested peers. For these students, adding extraneous illustrative features onto the diagram may have raised the required cognitive effort sufficiently (i.e., because it had both illustrative and diagrammatic features), so they may have preferred not to expend the needed effort and to disregard the figure altogether. This account could explain why diagrams were not beneficial for students with low interest in math, when an illustration was also present. In contrast, when there was no illustration present, fewer resources were needed to process the visual representation, so even students with low interest in math could benefit from the presence of the diagram.

Second, participants who valued mathematics more highly benefited more from diagrams than participants who valued mathematics less highly. Students who value mathematics highly are more likely to attempt to understand the mathematics and persist in the face of difficulty (e.g., Chouinard et al., 2007). The trigonometry problems we used were quite difficult, and students who valued math highly may have been more likely than those who valued math less highly to attempt to deeply understand the problems. In their efforts to do so, they may have relied heavily on the relevant spatial information in the diagrams. Thus, they experienced a greater positive effect of diagrams than did students who valued math less highly. Future studies could include measures of the extent to which students rely on the diagrams while problem solving.

Across these findings, we suggest that *interest* and *value* attitudes may have differential effects on students' processing of visual representations due to different, but overlapping, mechanisms. Students' interest in mathematics may influence their engagement, thereby altering the

cognitive resources they choose to devote to the task. Students' *value* attitudes may affect their motivation for mastering problems, which may influence whether they make use of supportive visual representations.

Overall, these findings are in line with the cognitive-affective theory of multimedia learning (e.g., Moreno, 2005, 2006; Moreno & Mayer, 2007), which holds that cognitive, motivational, and affective factors influence the information that people attend to and process in working memory. However, many questions remain unanswered. Do students with more positive attitudes devote more time to processing the diagrams or devote more effort to integrating diagrams with illustrations and text? Eye tracking or other process measures could be especially valuable in such investigations (Clinton, Cooper, Michaelis, Alibali, & Nathan, 2016; Park, Korbach, & Brünken, 2015).

Relations between problem-solving performance and problem evaluations

Overall, participants' evaluations of the problems were more favorable when diagrams or illustrations were present. Comparing problem evaluations and accuracy, we see that participants' positive evaluations of problems with diagrams generally aligned well with the positive effect of diagrams on accuracy in solving problems. However, the positive evaluations of diagrams did not depend on individual difference factors. Thus, the larger beneficial effect of diagrams for participants with more positive math value attitudes was not manifested in variations in their problem evaluations. Similarly, the lack of a benefit of diagrams when they were combined with illustrations for participants with less interest in math was not manifested in their problem ratings; they still rated problems with diagrams more favorably than problems without.

Participants' evaluations of problems with illustrations were less well aligned with their accuracy in solving those problems. Only participants with high mathematics ability solved problems more accurately when illustrations were present, but participants of all ability levels evaluated the problems more favorably when illustrations were present. In fact, participants of lower ability actually performed worse when illustrations were present, yet they evaluated problems with illustrations more favorably than problems without illustrations.

Although we did not ask students to predict their own performance, the findings raise the question of how well students would be able to predict how visual representations influence their performance. Students often have misconceptions about their own learning (Bjork, Dunlosky, & Kornell, 2013). Furthermore, students often believe that they learn more from multimedia materials than from text alone, regardless of whether or not this is the case (Serra & Dunlosky, 2010). In parallel to our findings, Serra and Dunlosky (2010) found that students accurately predicted that diagrams would improve their text comprehension; however, they inaccurately predicted that irrelevant illustrations would also improve their text comprehension. These authors attributed students' predictions to a general, undifferentiated belief about multimedia learning, namely, that visuals always help.

Further research is needed to address students' beliefs about learning with diagrams and illustrations and to ascertain how those beliefs interact with their attitudes and prior knowledge to affect learning.

This discrepancy between student favorability ratings and performance raises important issues for instructional design. The pattern of findings is particularly noteworthy in light of the motivation-based argument that textbook visuals help engage learners. If illustrations lead to greater engagement (or greater willingness to do more problems), they may contribute to long-term learning, even if they do not yield benefits for performance, or in the worst case, if they lead to poorer performance. To address this issue, the cumulative nature of experiences with visual representations over time will be important to consider.

Limitations and future directions

Some limitations of the present work must be acknowledged. First, although the type of problems we used was representative of problems students encounter in math classes, neither the problems nor the contexts were ones in which trigonometry would be used in daily life. This reduced authenticity may limit generalizability of our findings and could be more problematic for students less interested in mathematics, as they might have a harder time engaging with the problems.

Second, our individual difference measures were also limited in important ways. Self-reports of ability (such as ACT or SAT math scores) may be contaminated by reporting bias, and this may be especially prevalent among participants with lower ability (Cole & Gonyea, 2010). Our measure of mathematics attitudes provided separate measures of interest in mathematics and value placed on mathematics, but it did not measure other aspects of attitudes towards mathematics, such as expectancies about mathematics abilities or perceived task difficulty. Future research should consider the relevance of other individual difference measures in accounting for the varying effects of visual representations. In addition, future research should address the role of individual differences in other age groups and task domains.

Third, overall levels of performance in this study were not high, even for participants with high mathematics ability and positive attitudes towards mathematics. The problems were quite complex, and multiple steps were required to reach an accurate final answer, starting with mapping information from the problem to the visual representation. Participants needed to identify what quantity to solve for, figure out the steps needed to reach the solution, and correctly apply the trigonometric formulas to reach a final answer. Future research is needed to better understand the differential effects of visual representations on these different components of problem solving (Butcher, 2006). In particular, these analyses address the students' final answers that incorporate all of these steps of problem solving, rather than directly addressing students' solution processes. To reach a correct final answer, cognitive resources must be employed across the various components of the entire problem solving process.

Finally, it is worth noting that the contextualized illustrations we used differed from the majority of illustrations that are found in American mathematics textbooks (Mayer et al., 1995) because they also contained relevant mathematical content—they accurately depicted the spatial relationships described in the problems. In future research, we encourage researchers to compare such contextual illustrations with purely decorative illustrations, which do not provide mathematically relevant information (Magner et al., 2014). More generally, because there are diverse types of illustrations, it will be important to consider how different types and features of illustrations affect performance (Walkington, Clinton, & Mingle, 2016).

CONCLUSIONS

This research highlights the complexity inherent in using visual representations to support problem solving and learning. We observed an overarching beneficial effect of diagrams on problem solving, but the strength of this effect depended on participants' attitudes towards mathematics, and this effect was not observed for students with low interest in mathematics when illustrations were also present. In addition, we observed beneficial effects of contextual illustrations, but only for participants with high mathematics ability. Finally, participants' evaluations of problems with different types of visual representations did not always align with the effects of those representations on their performance. In particular, illustrations led to more favorable evaluations for participants across the range of ability, even though they were beneficial for problem solving only for participants with high ability. In future studies, rather than asking simply which types of visual representations serve learners better, it will be important to identify *how* learners with different ability levels and different attitudes towards mathematics use visual representations when solving problems. Identifying the processes students engage in during solving will be valuable, both for theoretical progress and for practical guidance about lesson design.

Our findings illustrate the challenges involved in making *a priori* predictions about how visual representations affect students' performance. From the cognitive, contextualization, and motivational perspectives, it is important to consider how individuals interact with multimedia materials, and these interactions depend on a complex web of factors, including factors unique to each individual. The present study demonstrates that both mathematics ability and attitudes towards mathematics moderate the effects of visual representations on problem-solving performance in a highly spatial mathematical domain. Thus, in predicting who benefits from visual representations in problem solving, both ability and attitudes matter.

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APPENDIX A

The following are problems as presented in the condition with both diagram and illustration. [Colour figure can be viewed at wileyonlinelibrary.com]

1) The parks department is putting a statue on a base. The statue is some distance away, and you are in a helicopter, eye level with its top. The angle of depression to the bottom of the statue (i.e., the top of the base) is 35 degrees. The height of the statue (without the base) is 50 feet. If someone were to stretch a string from the bottom of the base directly to you, it would be 100 feet long. How tall is the base?


35°

50 ft.

100 ft.
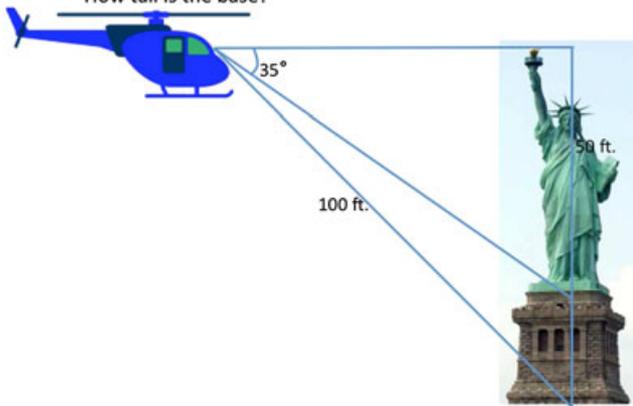
2) Your friend Jake is 5 feet tall. He is standing at the top of a 50-foot vertical cliff bordering a large lake. There is a blue sailboat coming toward him. The first time he measures, the angle of depression of the boat is 30 degrees. The second time he measures, the angle of depression of the boat is 45 degrees. Find the distance that the boat sailed between the two observations.

5 ft.

30°   45°

50 ft.
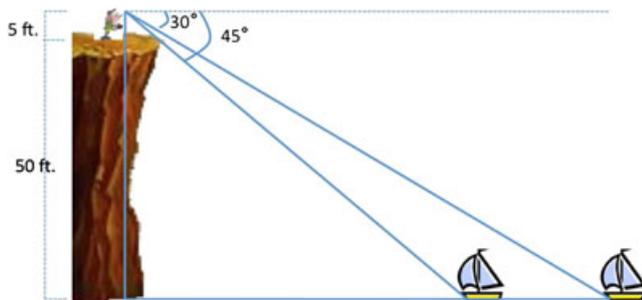
3) You are running around, flying your bright red kite, when all of a sudden, the kite gets snagged on the top of a house. The angle of elevation from the start of the kite string in your hand is 30 degrees. You run forward a little to investigate further, and notice that your angle of elevation is now 40 degrees, and the string length (held taut) from you to the top of the house is 60 ft long. How long was the string (held taut) when the kite got caught?

60 ft

30°   40°
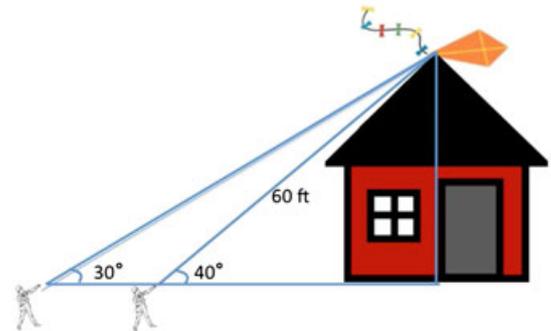
4) You lean a 100 ft ladder against a tower where Princess Marie is locked inside. Your ladder reaches the window. She tells you the the angle of depression of the ladder is 30 degrees. You see that is too steep to climb, so you extend the ladder's length to 150 ft and try again with the ladder's base now further from the tower. How far away is the base of your ladder from the tower?

30°

150 ft

100 ft
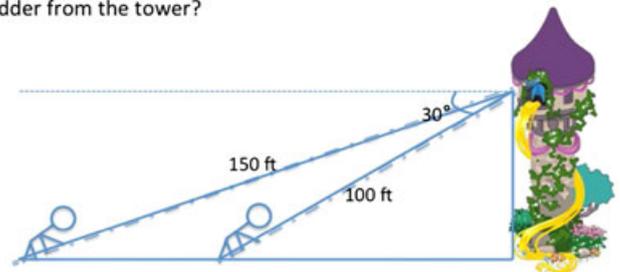
APPENDIX B

Table B1. Items loading on each factor in the factor analysis

Items loading on ability factor	
I find math confusing.	SD–SA
How good at math are you?	Not at all good, pretty good, and very good
I have a mathematical mind.	SD–SA
The prospect of having to learn something new in math makes me nervous.	SD–SA
Do you think math is hard?	Hard, OK, and easy
Items loading on interest factor	
How much do you like math?	A lot, a little, and not much
I find many math problems interesting.	SD–SA
Items loading on value factor	
How important is math to you?	Very important, a little important, and not important
Do you think math assignments are interesting?	Very interesting, a little interesting, and boring
Math is important in everyday life.	SD–SA
Excluded items	
If something about math puzzles me, I would rather be given the answer than have to work it out myself.	SD–SA
I prefer to work with symbols (algebra) rather than pictures (diagrams and graphs).	SD–SA

Note: Response choices for each item are given on the right. 'SD–SA' indicates response choices on a 5-point Likert scale ranging from 'strongly disagree' to 'strongly agree', with the midpoint labeled as 'neutral'.

Table B2. Correlations among items measuring math ability and attitudes

Factor	Item	1	2	3	4	5	6	7	8	9	10	11	12
Ability	1. I find math confusing.	—											
	2. How good at math are you?	**.57**	—										
	3. I have a mathematical mind.	**.63**	**.61**	—									
	4. The prospect of having to learn something new in math makes me nervous.	**.68**	**.45**	**.52**	—								
	5. Do you think math is hard?	**.65**	**.47**	**.5**	**.6**	—							
Interest	6. How much do you like math?	**.55**	**.46**	**.62**	**.56**	**.5**	—						
	7. I find many math problems interesting.	**.54**	**.44**	**.57**	**.53**	**.37**	**.78**	—					
Value	8. How important is math to you?	**.34**	.2	**.39**	**.38**	**.21**	**.64**	**.55**	—				
	9. Do you think math assignments are interesting?	**.51**	**.22**	**.4**	**.43**	**.34**	**.64**	**.67**	**.44**	—			
	10. Math is important in everyday life.	**.45**	**.25**	**.52**	**.45**	**.27**	**.54**	**.57**	**.54**	**.39**	—		
Excluded	11. If something about math puzzles me, I would rather be given the answer than have to work it out myself.	**.41**	**.31**	**.46**	**.4**	.2	**.46**	**.49**	**.37**	**.36**	**.39**	—	
	12. I prefer to work with symbols (algebra) rather than pictures (diagrams and graphs).	.06	.08	.08	.09	0	.01	−.07	−.04	−.08	−.07	.14	—

Note: Bolded correlations are significant at $p < .05$.