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To cite this article: Martha W. Alibali , Mitchell J. Nathan , Matthew S. Wolfgram , R. Breckinridge Church , Steven A. Jacobs , Chelsea Johnson Martinez & Eric J. Knuth (2014) How Teachers Link Ideas in Mathematics Instruction Using Speech and Gesture: A Corpus Analysis, *Cognition and Instruction*, 32:1, 65-100, DOI: [10.1080/07370008.2013.858161](https://doi.org/10.1080/07370008.2013.858161)

To link to this article: <http://dx.doi.org/10.1080/07370008.2013.858161>



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# How Teachers Link Ideas in Mathematics Instruction Using Speech and Gesture: A Corpus Analysis

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This research investigated how teachers express links between ideas in speech, gestures, and other modalities in middle school mathematics instruction. We videotaped 18 lessons (3 from each of 6 teachers), and within each, we identified *linking episodes*—segments of discourse in which the teacher connected mathematical ideas. For each link, we identified the modalities teachers used to express linked ideas and coded whether the content was new or review. Teachers communicated most links multimodally, typically using speech and gestures. Teachers' gestures included depictive gestures that simulated actions and perceptual states, and pointing gestures that grounded utterances in the physical environment. Compared to links about new material, teachers were less likely to express links about review material multimodally, especially when that material had been mentioned previously. Moreover, teachers gestured at a higher rate in links about new material. Gestures are an integral part of teachers' communication during mathematics instruction.

There is great variability in what students learn from classroom instruction. Many factors are potentially relevant to understanding variations in students' learning. In this research, we focus on one such factor: the nature of teachers' communication about key aspects of the instructional material. We focus in particular on teachers' communication about relationships between ideas in mathematics lessons, because such relationships are often challenging for students to grasp but also central to mathematical understanding. Our primary goal in this work was to document how teachers express links between ideas in speech, gestures, and other modalities in naturalistic classroom instruction in middle school mathematics.

### LINKING IDEAS IN MATHEMATICS

Mathematical entities, such as quantities, relationships, and procedures, can be represented in a variety of ways. For example, a function can be represented using an equation, a graph, or a table of values. The ability to work with and translate among different external representations of mathematical ideas is an overarching process standard described by the National Council of Teachers of Mathematics in the *Principles and Standards for School Mathematics* (PSSM). Students are expected to be able to "select, apply, and translate among mathematical representations to solve problems" (National Council of Teachers of Mathematics [NCTM], 2000, p. 67).

Mathematical entities gain meaning through their links to other, related mathematical entities. Some of these links involve connections among different mathematical representations. For example, a linear function gains meaning via the links between its symbolic representation and its graphical representation. Other links involve connections within mathematical representations. For example, a point on a graph gains meaning via its relations to other components of the graph, such as the axes and the origin. Still other links involve connections among ideas at more elemental levels. For example, the slope of a line in a graph may gain meaning via its relation to the slope of another line in a different graph. As a second example, the operation of division gains meaning through its relations to multiplication and subtraction.

Many researchers have argued that understanding relationships among mathematical representations, and among mathematical ideas in a broader sense, is fundamental to a deep, conceptual understanding of mathematics. For example, in their textbook, *Making Sense: Teaching and Learning Mathematics with Understanding*, James Hiebert and colleagues argue, "we understand something if we see how it is related or connected to other things we know" (Hiebert et al., 1997, p. 4). In defining conceptual understanding, they further state, "Understanding can be characterized by the kinds of relationships or connections that have been constructed between ideas, facts, procedures, and so on" (p. 15). In the mathematics cognition literature, understanding connections is one of the most commonly proffered definitions of conceptual knowledge (Crooks & Alibali, 2013a, 2013b). Of course, students may sometimes rely on knowledge of connections without deeply grasping the meaning of those connections. For example, Kaput (1998) noted that students sometimes operate on "representational islands" that are not connected to the "mainland" of experience. Nevertheless, by and large, there is agreement that grasping connections among mathematical ideas is valuable for understanding and using mathematics.

Even though connections are essential to mathematical understanding, they are often challenging for students to understand, as many studies—not only in mathematics education, but also in cognitive science—have shown (e.g., Ainsworth, 1999; Brenner et al., 1997; Gilmore, 2006;

Janvier, 1987; Knuth, 2000; Kurtz, Miao, & Gentner, 2001; Leinhardt, Zaslavsky, & Stein, 1990; Lesh, Post, & Behr, 1987). Despite students' difficulties, understanding connections is one of the process standards in the PSSM; students are expected to understand "how mathematical ideas interconnect and build on one another to produce a coherent whole" (NCTM, 2000, p. 64).

Because connections are integral to students' understanding of mathematics, teachers use a variety of instructional techniques to make connections during instruction including: sequencing topics so that connections are highlighted; comparing related procedures or concepts; eliciting self-explanations; making analogies; and providing direct information about connections. In this research we examine how middle school teachers communicate about connections in classroom instruction in mathematics, including connections within and among representations and connections at more elemental levels.

Our focus is not on whether specific pedagogical connections influence student thinking or learning but rather on how teachers communicate about connections. Of course, not all connections are inherently good or equally valuable. Some connections may be more productive than others, and in some cases teachers may make connections that are pedagogically problematic or even incorrect. In this research, we did not attempt to assess the pedagogical value of the specific connections that we observed teachers make. We also did not attempt to assess whether students grasped the particular connections that were the target of instruction. Instead, our aim was to document the range of ways that teachers express connections among ideas during classroom mathematics instruction in middle school. In our view, there is a need for careful research on the kinds of behaviors teachers engage in during authentic instruction.

We chose middle school because it is a period when instruction focuses on both arithmetic and algebraic reasoning and the connections between them. Middle school marks a significant transition from the concrete, arithmetic reasoning of elementary school mathematics to the increasingly complex, abstract algebraic reasoning required for high school mathematics and beyond. Moreover, the central mathematical ideas of middle school are as difficult conceptually as any ideas in the K–12 mathematics curriculum (National Research Council [NRC], 2000). Lastly, during middle school mathematics instruction, multiple representations of mathematical ideas are commonplace, and connections between representations and within individual representations are often the focus of instruction.

## GESTURE AS AN INTEGRAL ELEMENT OF INSTRUCTIONAL COMMUNICATION

Past research has shown that mathematics teachers use a range of modalities to represent and communicate mathematical ideas during instruction. These modalities include speech, writing, drawing, manipulatives, and gestures. Nonverbal representations of ideas are common in mathematics instruction. For example, Flevares and Perry (2001) studied first-grade mathematics teachers' classroom instruction on place value and found that the teachers used approximately five to seven nonverbal representations (including pictures, symbols, concrete objects, and gestures) per minute of instruction.

In this work, we focus on how teachers communicate about links between ideas not only with speech but also nonverbally. We focus specifically on how teachers use gestures in making connections. Gestures can be defined as spontaneous movements, typically of the hands and

arms, that people produce when speaking (McNeill, 1992). Over 10 years ago, Roth (2002) noted that there was “very little educational research concerned with the role of gestures in learning and teaching” (p. 365). In the years since, there has been a surge in research on gesture in instructional settings, conducted by researchers from diverse theoretical perspectives using diverse methodological approaches (e.g., Edwards, 2009; Goldin-Meadow & Singer, 2003; Kim, Roth, & Thom, 2011; Nemirovsky, Rasmussen, Sweeney, & Wawro, 2012; Núñez, 2005; Reynolds & Reeve, 2002; Richland, Zur, & Holyoak, 2007; Singer, Radinsky, & Goldman, 2008). One likely reason for this surge is that gestures are ubiquitous in instructional settings. For example, in the lessons studied by Flevares and Perry (2001), teachers used gestures more frequently than they used other nonverbal representations (such as pictures, symbols, and objects), and moreover, when teachers combined representations, they nearly always used gestures. Gestures are readily available at all times, so it is not surprising that teachers produce them regularly.

Our focus on teachers’ gestures is informed by the large body of research showing that gestures contribute to communication in a range of settings (for reviews, see Hostetter, 2011; Kendon, 1994). With respect to instruction, there is growing evidence that lessons with gestures can be more beneficial for students’ learning than matched lessons without gesture. In many cases, students show deeper learning (i.e., better uptake of instructional information, new forms of reasoning, generalization to new problem types, retention of knowledge) from lessons with gestures. For example, one study investigated first-grade students learning about Piagetian conservation of quantity from videotaped lessons. Among participants who were native English speakers, 91% showed deep learning (i.e., added new same judgments) from a speech-plus-gesture lesson, compared to 53% from a speech-only lesson. Among participants who were Spanish speakers with little English proficiency, 50% learned from the speech-plus-gesture lesson, compared to 20% from the speech-only lesson (Church, Ayman-Nolley, & Mahootian, 2004). Another study examined eighth-grade students’ learning from a videotaped lesson about polynomial multiplication and found that students displayed greater learning (i.e., greater success multiplying polynomials at posttest) from a speech-plus-gesture lesson than from either a speech-only lesson or a lesson in which the referents of speech were highlighted in yellow as the instructor spoke about them (Bem et al., 2012). Similar findings have also been reported with other concepts and age groups (Cook, Duffy & Fenn, 2013; Perry, Berch, & Singleton, 1995; Singer & Goldin-Meadow, 2005; Valenzeno, Alibali, & Klatzky, 2003).

Clearly, teachers’ gestures can have a substantial impact on students’ learning from instruction. However, the speech-only control conditions in the experimental studies described above lack ecological validity because it is extremely unlikely that a real teacher would produce no instructional gestures at all. In order to better understand the role of teachers’ gestures in authentic classroom communication, it is important to consider how teachers actually gesture in naturalistic settings. In particular, it would be valuable to document the range of ways in which teachers use gestures to communicate essential instructional content such as connections among ideas.

## GESTURES MANIFEST EMBODIED COGNITIVE PROCESSES AND SITUATED COMMUNICATIVE PRACTICES

To understand the importance of gesture in instructional communication, it is necessary to consider both why speakers produce gestures and how gestures might play a role in communication.

Our thinking on this issue draws from multiple theoretical perspectives, including embodied cognition, situated cognition, and contemporary psycholinguistic accounts of gesture production and comprehension.

### Why Speakers Produce Gestures

Although the details of specific theories differ, many contemporary psycholinguistic accounts of gesture production view gesture as arising from spatial or motoric mental representations that are deeply interconnected with the linguistic mental representations that underlie speech (e.g., Kita & Özyürek, 2003; McNeill, 1992). One recent formulation of this perspective is the *Gesture as Simulated Action* framework (Hostetter & Alibali, 2008). This framework holds that both gesture and speech are based on simulated actions and perceptual states, which activate premotor and motor areas of the brain. Speakers sometimes fail to inhibit this motor activation, and they produce gestures as a result. From this perspective, speakers produce gesture as a natural part of their effort to communicate intended meanings that have perceptual and action-based elements.

Simulated actions and perceptual states play a central role in the *Gesture as Simulated Action* framework. This view is aligned with the embodied cognition perspective, which holds that human cognitive processes have their roots in action and perception (e.g., Barsalou, 2008; Glenberg, 2010). Thus, when speakers' gestures represent elements of meaning, those depictive gestures provide evidence for the nature of the simulated actions or perceptual states that underlie speakers' communication (see also Nathan & Johnson, 2012). As one example, Edwards (2009) describes a teacher who produced a series of cutting motions when talking about fractions as parts of a pie. From her gestures, we can infer that this teacher mentally simulated the action of cutting a pie.

As a second example, Marghetis, Bergen, and Núñez (2012) describe a student talking about adding two identical numbers. She said, "Because you add the same numbers," while she produced a gesture in which she brought her hands in grasping hand shapes together in front of her body, evoking the combination of two collections or masses. Marghetis et al. (2012) interpret this gesture as evidence for the conceptual metaphor *arithmetic is collecting objects*. In this example, the conceptual action of adding numbers is grounded in the physical action of collecting objects, suggesting that even abstract mathematical notions are rooted in the actions of the body. From the perspective of the *Gesture as Simulated Action* framework, the student simulates the action of collecting objects and expresses this simulation in gestures.

However, not all gestures represent simulated actions and perceptions. Some gestures indicate objects or locations in the physical world via pointing. From an embodied cognition perspective, pointing gestures manifest the grounding of cognition in the physical environment. Pointing gestures have meaning because they index objects and locations in the physical world (see Glenberg & Robertson, 1999, for similar arguments about words). Pointing gestures often refer directly to the objects or inscriptions they indicate (e.g., pointing to an equation written on the blackboard to refer to that equation). At other times, pointing gestures refer indirectly by using the indicated objects, inscriptions, or locations to stand in for other related objects, inscriptions, or locations (e.g., pointing to an equation on the board to refer to a similar equation that was discussed earlier in the lesson).

In brief, we argue that speakers produce depictive gestures because they simulate actions and perceptions that are part of speakers' intended meanings, and speakers produce pointing gestures

as a consequence of the grounding of cognition in the physical environment (Alibali & Nathan, 2012).

### How Gestures Communicate

The embodied cognition perspective also has implications for understanding gesture's role in communication. We suggest that listeners may better grasp speakers' embodied meanings when those meanings are expressed in gestures. First, speakers' gestures can help listeners to grasp speakers' referential intentions by indexing objects and inscriptions in the socially shared environment. Second, speakers' gestures can help listeners to grasp speakers' semantic intentions by helping listeners themselves to simulate relevant actions and perceptions.

Pointing gestures communicate directly by indexing the linguistic objects with which they occur (i.e., words and phrases) to objects, inscriptions, and locations in the physical world (Glenberg & Robertson, 1999). Speakers often use pointing gestures to index objects or inscriptions that are present in the socially shared environment. In addition, as noted above, speakers sometimes use pointing gestures to index nonpresent objects or inscriptions. These pointing gestures—both literal and nonliteral—presumably help listeners to index the same referents that speakers have in mind. As such, pointing gestures help listeners to grasp speakers' referential intentions.

Depictive gestures that express simulated actions and perceptions may communicate by helping listeners to construct corresponding action and perception simulations in their own minds (Alibali & Hostetter, 2011). Studies of action understanding suggest that observing actions leads to covert motor simulations of those same actions (e.g., see Prinz, 1997, for review). Moreover, neuroscientific studies suggest that when people observe actions, there is corresponding activation in premotor areas (see Jeannerod, 2001; Rizzolatti, Fogassi, & Gallese, 2001, for reviews). These same processes may apply when perceiving and interpreting others' gestures, because gestures are a form of action (Alibali & Hostetter, 2011; Ping, Goldin-Meadow & Beilock, 2103). Thus, seeing a speaker's depictive gestures may help listeners to generate action and perception simulations that correspond with those of the speaker. In this way, speakers' depictive gestures help listeners to grasp speakers' semantic intentions.

A situated view of communication views gesture as one of several semiotic systems, each having different properties, that are simultaneously deployed in human interaction (e.g., Goodwin, 2000). This view is compatible with the embodied perspective described above, which focuses on speakers' intended meanings. A situated perspective enriches the embodied view by placing a sharper focus on features of the socially shared physical environment (including relevant artifacts and inscriptions), the participants' roles in the interaction, and the multiple interacting semiotic systems at play (e.g., talk, action, gesture, and so forth). These factors are all relevant from the embodied perspective as well.

The situated perspective has been fruitfully applied in analyzing interaction in mathematics classrooms (e.g., Arzarello, 2006; Nemirovsky & Ferrara, 2009; Radford, Bardini, & Sabena, 2007). Gesture can be viewed as one of several semiotic resources that teachers and learners may draw on in developing, refining, and clarifying ideas (e.g., Marrongelle, 2007; Rasmussen, Stephan, & Allen, 2004). Illustrating this point, Arzarello, Paola, Robutti, and Sabena (2009) show that teachers sometimes adopt and repeat students' gestures as part of what they term a

*semiotic game*, in which teachers guide students toward more appropriate mathematical language and actions. Along similar lines, Keene, Rasmussen, and Stefan (2012) document a “chain” of gestural signs used by teacher and students over a series of lessons and argue that these gestures help scaffold students’ understanding of a challenging mathematical concept.

In light of this work on gestures as a semiotic resource for communication in classroom settings, it is not surprising that teachers’ gestures make a difference in students’ learning from instruction (e.g., Goldin-Meadow, Kim, & Singer, 1999; Singer & Goldin-Meadow, 2005; Valenzeno et al., 2003). However, as noted above, most experimental studies of instructional gestures have involved videotaped lessons with limited ecological validity. On the other hand, most studies of gesture “in the wild” of real classrooms have involved analyzing only a small number of compelling cases. In this study, we seek to bridge these research traditions by presenting a quantitative analysis of teachers’ use of gestures in a corpus of mathematics lessons and by presenting representative examples in detail. We focus in particular on teachers’ use of gestures to connect ideas during mathematics instruction.

### GOALS OF THE PRESENT STUDY

Past research has described some cases in which teachers used gestures to connect ideas (Alibali, Nathan, & Fujimori, 2011; Nathan & Alibali, 2011; Richland et al., 2007). However, to date there has been no systematic study of how teachers use gestures to link ideas in mathematics instruction. Thus, the primary goal of this research was to document how teachers express links between ideas in speech, gestures, and other modalities in naturalistic classroom instruction in middle school.

First, we wished to know how often teachers explicitly performed such linking in their instruction; this is of interest in part because of its relevance to some of the goals for mathematics education stated by the PSSM (NCTM, 2000) and the recent Common Core standards (National Governors Association Center for Best Practices, 2010). For example, according to the PSSM, “A coherent curriculum effectively organizes and integrates important mathematical ideas so that students can see *how the ideas build on, or connect with, other ideas* [emphasis added]” (NCTM, 2000, p. 16). The PSSM also recognizes the important role of classroom communication in helping build connections among ideas: “Classroom discourse and social interaction can be used to promote the recognition of connections among ideas and the reorganization of knowledge” (NCTM, 2000, p. 21). To foreshadow our results, we found that *linking episodes* (i.e., segments of discourse in which teachers sought to link ideas) occurred quite frequently during middle school mathematics instruction (on average, more than 10 per hour of instruction), and therefore we found it of interest to analyze teachers’ communication during linking episodes in greater detail. We were particularly interested in how frequently teachers used multiple modalities (e.g., gesture and speech) when connecting ideas during instruction. In light of past research, we predicted that this would occur regularly.

We also sought to identify the range of different ways in which teachers might utilize gestures when connecting ideas during instruction. We focused on two dimensions of teachers’ gestures to linked ideas that, based on past work, seemed potentially relevant: the types of gestures (in particular, pointing vs. depictive gestures) and the timing of the gestures.

As described above, people commonly simulate actions and perceptual states using depictive gestures, and they also commonly refer to objects, inscriptions, or locations in the physical world via pointing gestures. We predicted that teachers would use both of these types of gestures in communicating about mathematical ideas; for example, they might point to elements of an inscription to refer to those elements, or they might represent a geometric shape via depictive gestures to evoke a mental image of that shape in their students' minds.

People can use gestures to indicate or represent more than one thing at a time (e.g., by using both hands simultaneously or by using two or more fingers from one hand simultaneously). When multiple ideas are presented close together in time, students stand to benefit, as past research on multimedia learning has suggested (Mayer, 2009). Therefore, we predicted that teachers might, in some cases, use gesture to simultaneously connect related ideas. Alternatively, teachers might use gestures to present first one idea and then another—that is, to connect related ideas sequentially.

Thus, in documenting the ways that teachers use gesture in communicating about links between ideas, we identified cases in which links were made by referring to external representations of ideas via pointing and cases in which links were made by evoking ideas via depictive gestures. Further, we also identified cases in which links were made using sequential gestures and cases in which links were made using simultaneous gestures.

A secondary goal of this work was to systematically examine whether the ways that teachers connect ideas vary as a function of whether the material is new to the students or whether it had been previously introduced. In a previous investigation of a single lesson from a middle school mathematics teacher, Alibali and Nathan (2007) found that the teacher used more gestures when teaching content that was new to the students than when teaching content that was review. However, this finding was limited to a single teacher, so it bears replication in a broader sample. The finding makes sense in light of other research showing that speakers rely on gesture to communicate spatial information that is not part of the *common ground*, or the knowledge that they share with their interlocutors (Holler & Stevens, 2007). For the present study, therefore, we predicted that teachers would express links in multiple modalities (i.e., in speech and gesture together) more frequently for connections that are new to the students than for connections with which students are already familiar (i.e., that are review).

In brief, our goals in this research were (a) to document how teachers express links between ideas in gestures, speech, and other modalities in naturalistic classroom instruction in beginning algebra, and (b) to explore whether the ways in which teachers express links between ideas vary depending on whether the material is new or review.

## METHODS

### Participants

Participants were six middle school teachers (one sixth-grade, two seventh-grade, and three eighth-grade teachers) from a mid-sized city in the American Midwest. Five of the six taught in the city's public school district (at three different middle schools) and one taught in a parochial school. Four were female, and two were male.

TABLE 1  
Grade Level and Lesson Topics for Each Set of Lessons

<i>Teacher</i>	<i>Gender</i>	<i>Grade</i>	<i>Lesson Topics</i>
A	M	8	Defining polynomial and the degree of a polynomial; writing polynomials in standard form; multiplying a polynomial by a monomial; using algebra tiles to represent an area model strategy of multiplying a polynomial by a monomial
B	F	7	Connecting what students had learned about divisibility rules with identifying prime and composite numbers; comparing the process of prime factorization of a number to that of a monomial; applying the process of prime factorization to simplifying fractions
C	F	7	Investigating the relationships among scale factor, measurements, and percentages; using algebraic rules to produce similar figures on a coordinate grid; using corresponding angle and side lengths to contrast similar figures with nonsimilar figures
D	M	8	Using “recursive routines” to solve word problems involving linear rates
E	F	6	Using strategies to estimate the product of mixed fractions; developing procedures for multiplying mixed fractions
F	M	8	Understanding the difference between “percentage change” and “growth factor”; determining growth factors (whole and fractional) from an exponential population model; developing equations to represent exponential population models and predict future values; interpreting graphs of exponential relationships

### Source of Data

For each teacher, we videotaped three entire lessons from consecutive class days. The research team did not preselect the lesson topics; we scheduled our videotaping to accommodate teachers’ preferences. Lesson topics are listed in Table 1. Lessons ranged from 34 to 61 minutes in length ( $M = 49$  min).

Each teacher completed a prelesson survey for each lesson. In this survey, teachers were asked to describe (a) what concepts and/or procedures the teacher expected students to learn during the lesson, (b) what new mathematical representations or notations the teacher planned to introduce to students for the first time during the lesson (if any), (c) what the teacher’s expectations were for the students during the lesson, and (d) what difficulties the teacher expected students to have during the lesson.

We also interviewed each teacher briefly after each lesson (see Appendix for interview materials). In the interview, teachers again identified which portions of the lesson were new material and which were review.

### Video Analysis

We first viewed the entire lesson and then prepared a lesson summary that described what occurred during the lesson and identified the mathematical representations that figured prominently in the lesson. We also prepared a complete verbal transcript of each videotaped lesson. We reviewed several of the lessons in detail during data workshops with our research team, and we noted that each lesson contained several segments of discourse during which the teacher sought to connect

mathematical ideas. These episodes typically included some focus on each of the linked ideas as well as some statement or expression of the connection between them.

Building on this qualitative work, we then conducted a quantitative analysis of teachers' linking in the full set of 18 lessons. Our analysis involved both the transcript and the video, and it proceeded in five steps described in detail in this section. These steps were:

1. We identified segments of discourse in which teachers sought to link ideas, referred to as *linking episodes*.
2. In each linking episode, we identified the portion of the discourse that directly expressed the relationship between ideas, referred to as the *target link*.
3. We described the modalities used to indicate or describe the linked ideas in the target links, including speech, gesture, writing, and so forth.
4. We assessed whether the gestures used to link ideas were produced *sequentially* (i.e., first one and then the other) or *simultaneously* (i.e., both at the same moment).
5. We classified whether each linking episode presented new material or reviewed previously taught material.

*Identifying Linking Episodes and Target Links (Steps 1 and 2).* On an initial pass through the video, we identified linking episodes, defined as segments of discourse in which the teacher sought to link two (or sometimes three or more) mathematical ideas. These episodes typically extended over several lines of transcript and provided a context for the link being made. For each linking episode, the coders also identified the specific ideas that were linked in the episode. In some cases, a teacher made a general link between two ideas and then fleshed out that general link by connecting specific components of the two ideas. Such cases were counted as single linking episodes. For example, one teacher made a general link between two columns in a table of values (see Figure 1) and then elaborated on that general link by connecting individual cells in the two columns. For analysis, this episode was counted as a single link. Thus, our counts reflect the larger episodes in which general links between ideas or representations were made rather than more specific links among subcomponents of those ideas or representations.

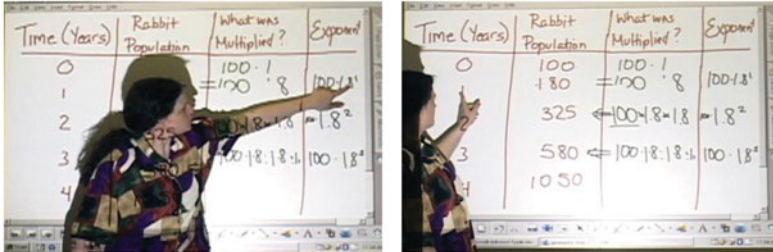
We construe mathematical ideas broadly to include inscriptions that are traditionally used in mathematics (e.g., equations, graphs, tables), and also to include verbal expressions of mathematical representations or entities, pictorial and gestural depictions of mathematical representations or entities (e.g., a diagram drawn on the board or in the air), and concrete manipulatives and other objects (real or imagined) used to represent mathematical entities (e.g., objects such as a carpeted room referred to in a discussion of area). We focused not only on links across mathematical representations (e.g., links between an equation and a table) but also on links within a single representation (e.g., links between one column within a table and another column within that same table) and on links between ideas at more elemental levels (e.g., links between a side of one triangle and the corresponding side of a second triangle).

Within each linking episode, we then identified the target link, which was essentially the heart of the linking episode. We defined the target link as the portion of the linking episode in which the relationship between ideas was directly stated or implied. Most target links were between one and six lines of transcript.

V1. So [this one] tells me the [first year],

G1. Left hand point to 1 as exponent

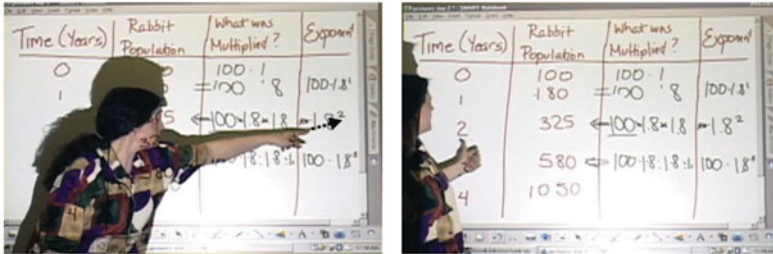
G2. Left hand point to 1 in Years column



V2. [this two] tells me the [second year].

G3. Left hand point to 2 as exponent

G4. Left hand point to 2 in Years column



V3. The [exponent] matches the [year]

G5. Left hand point to Exponent column

G6. Left hand point to Years column

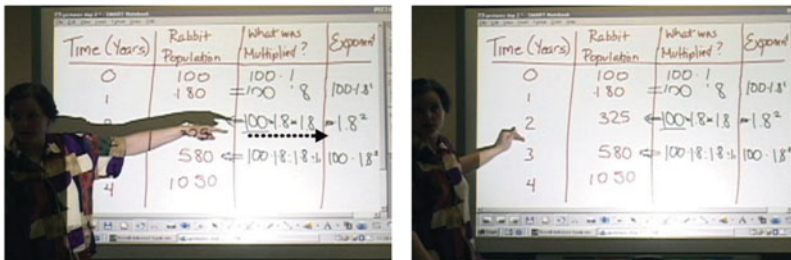


FIGURE 1 Linking episode in which the teacher expresses both linked ideas in both speech and gesture. Square brackets identify the words during which each gesture was produced. (Teacher F) (Color figure available online). (Continued)

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V4. so we don't have to do the [x minus one]].

G6. Left hand point to Years column, held from previous line through this entire line

G7. Right hand sweeps to right to represent "x minus one"

Time (Years)	Rabbit Population	What was Multiplied?
0	100	1
1	180	1.8
2	325	1.8
3	580	1.8
4	1050	1.8

FIGURE 1 (Continued). (Color figure available online).

The linking episodes and target links within them were first identified by an initial coder and then reviewed and checked by a second coder. For each linking episode the coders discussed and reached consensus on the identity of the particular linked ideas and on the boundaries of the linking episode and target link.

*Coding the Modalities Used Within Target Links (Step 3).* Next we identified the expressive modalities that teachers used in each target link. These included speech, gestures, writing, drawing, and actions on manipulatives.

For each link in which one or more of the linked ideas were expressed in gestures, we further coded each of the teacher's gestures into one of four categories, building on the gesture classification system originally developed by McNeill (1992). *Pointing* gestures are gestures that indicate objects, locations, or inscriptions, usually with an extended finger or hand (e.g., indicating a number in an equation on the board by pointing to that number with the index finger). *Depictive* gestures are gestures that depict aspects of semantic content directly, via hand shape or motion trajectory, either literally (e.g., representing a triangle by tracing it in the air, or representing an angle by holding the two hands palms facing, heels together) or metaphorically (e.g., cupping both hands as if to hold an idea, which manifests the metaphor *ideas are objects*).<sup>1</sup> *Beat* gestures are motorically simple rhythmic movements that do not carry semantic content but that often align with the prosody or discourse structure of speech. *Writing* gestures were defined as writing or drawing actions that were integrated with speech in the way that hand gestures are typically integrated with speech but that were produced while holding a writing instrument (usually chalk or marker) and that involved writing to indicate or illustrate the content of the accompanying speech (e.g., underlining an equation on the board while saying "this equation").

<sup>1</sup>Gestures that depict their referents literally are often referred to in the literature as iconic gestures, and those that depict their referents metaphorically are often referred to as metaphoric gestures (e.g., McNeill, 1992). In this article, we collapse these two categories into the general category of depictive gestures.

These writing gestures could readily be distinguished from writing produced as a functional act (i.e., writing in order to get something on the board) in a manner not aligned with the accompanying speech.

*Coding Whether Gestures in Target Links Were Sequential or Simultaneous (Step 4).* For each link in which both or all of the linked ideas were expressed in gestures, we coded whether teachers gestured to the linked ideas sequentially (i.e., first gestured to indicate or represent one idea, then gestured to indicate or represent another idea) or simultaneously (i.e., gestured to indicate or represent one idea with one hand and another idea with the other hand, at the same moment). Any set of gestures that included some simultaneous gestures was coded as simultaneous.

*Identifying Whether Linking Episodes Were New Material or Review (Step 5).* Each linking episode was also coded in terms of whether it conveyed material that was new to the students or material that was review for the students. These judgments were made based on information gleaned from the prelesson survey, from the nature of the lesson segment during which the link was produced (i.e., warm-up, homework discussion, main lesson activity), and from remarks or statements that teachers made during the lessons and in the postlesson interviews. Links made during warm-up exercises and during homework discussion were considered review unless other information (from the prelesson survey or from teacher comments) indicated otherwise. Teachers frequently made comments during the lessons that indicated the status of the material. For example, remarks such as “Yesterday we discussed . . .” or “Who remembers how. . .” indicate review material; remarks such as “I’m going to show you a new way to . . .” indicate new material.

For each linking episode, we also noted whether the specific connection that was the focus of the link was being mentioned for the first time within that lesson or for the second (or later) time within the lesson. Thus, links were classified into four categories: (a) new, first mention; (b) new, later mention; (c) review, first mention; or (d) review, later mention.

### Reliability of Coding

To establish reliability on identifying the modalities in which each of the linked ideas was expressed, a second coder recoded a subset of 16 target links, which included 34 linked ideas. Agreement regarding the modalities in which ideas were expressed was 96% ( $N = 34$ ).

To establish reliability on identifying and classifying gestures, a second coder recoded a subset of 17 target links. For identifying gestures from the stream of manual action, agreement was 90% ( $N = 120$ ). For classifying gestures as pointing, depictive, beat, or writing gestures, agreement was 86% ( $N = 109$ ).

For links in which both ideas were expressed in both speech and gesture, a second coder also coded whether teachers’ gestures were purely sequential or whether they also included simultaneous linking of the related ideas. The coders agreed in 92% of cases ( $N = 13$ ).

## RESULTS

### How Often Did Teachers Link Ideas in Their Instruction?

Teachers varied in the frequency with which they communicated links between ideas in their lessons. The average number of linking episodes per lesson across the six teachers was nine ( $SD = 4.73$ , range 5–15.7). Considered in terms of links per unit time, teachers produced an average of 10.8 links per hour of instruction ( $SD = 4.81$ , range 5.9–18.3). Because the lesson content varied across teachers, we cannot determine whether the frequency of linking in the lessons we observed depends on variations in lesson content or on individual differences in teachers' instructional styles. Some topics may have afforded more opportunities to link ideas than did other topics.

### In What Modalities Did Teachers Express Linked Ideas?

For each target link, we identified the modalities in which each of the linked ideas was expressed: (a) speech alone; (b) speech and functional uses of either writing or drawing (or both); (c) speech and gesture (including pointing, depictive gestures, or writing gestures); (d) speech and action on a manipulative; (e) speech and writing or drawing and gesture; or (f) gesture alone. Because beat gestures do not convey semantic content, beat gestures were not considered in determining the modalities in which the linked ideas were expressed.

Given that links between ideas are often central to mathematics lessons, we hypothesized that teachers would tend to express the linked ideas in multiple modalities rather than expressing some of the linked ideas in a single modality such as speech alone. Table 2 classifies the target links produced by each teacher (summed across all three lessons for each teacher) in terms of the modalities in which the linked ideas were expressed. As seen in the table, in the vast majority of target links, both of the linked ideas were expressed multimodally, typically in speech and gesture, sometimes speech with writing and/or drawing, and sometimes in speech, gesture, and writing or drawing. Each of the six teachers expressed both (or all) of the linked ideas multimodally in a majority of their target links ( $M = 90\%$  of linking episodes, range 65–100%).

We then examined whether teachers were more likely to produce multimodal links than unimodal links. We compared the proportion of links that were expressed multimodally against .5 (i.e., the value expected based on the null hypothesis that unimodal and multimodal links would be equally likely). The observed proportion of multimodal links was significantly greater than .5,  $t(5) = 6.29$ ,  $p = .001$ . Thus, the data are incompatible with the idea that teachers are equally likely to use multimodal and unimodal links; instead they suggest that teachers tend to express links between mathematical ideas multimodally.

### Links in Which Both Ideas Were Expressed in Both Speech and Gesture

Figure 1 presents a representative example of a linking episode in which a teacher expressed both of the linked ideas in both speech and gesture. In this example, and in all other examples in this paper, each line of the verbal transcript is numbered (as V1, V2, V3, etc.), and the teacher's

TABLE 2  
Target Links Produced by Each Teacher, Summed Across All Three Lessons for Each Teacher. Links are Classified in Terms of the Modalities in Which Each of the Linked Ideas was Expressed

<i>Modalities in Which Linked Ideas Expressed</i>	<i>Teacher A</i>	<i>Teacher B</i>	<i>Teacher C</i>	<i>Teacher D</i>	<i>Teacher E</i>	<i>Teacher F</i>	<i>Total</i>
<i>Links in Which One or More Ideas Were Expressed in a Single Modality</i>							
Both/all in speech alone	0	0	0	0	2	0	2
Speech alone <i>and</i> speech+gesture	5	1	1	0	2	1	10
Speech alone <i>and</i> speech+writing	0	0	0	0	1	0	1
Speech alone <i>and</i> speech+gesture+writing	0	0	0	0	2	1	3
Gesture alone <i>and</i> speech+gesture <i>or</i> speech+writing <i>or</i> speech+gesture+writing	0	0	0	0	0	2	2
Total (single-modality links)	5	1	1	0	7	4	18
<i>Links in Which Both (or All) Ideas Were Expressed in Multiple Modalities</i>							
Both/all in speech+writing	0	0	1	0	0	1	2
Both/all in speech+gesture	11	11	15	15	5	30	87
Both/all in speech+action	0	0	1	0	0	0	1
Both/all in speech+writing+gesture	4	2	2	0	1	3	12
Speech+gesture <i>and</i> speech+writing	7	1	0	1	2	3	14
Speech+gesture <i>and</i> speech+gesture+writing	15	0	2	2	2	5	26
Speech+writing <i>and</i> speech+gesture+writing	0	0	0	0	0	1	1
Speech+writing <i>and</i> speech+gesture+action	1	0	0	0	0	0	1
Total multimodal links	38	14	21	18	10	43	144

*Note.* Writing includes both writing and drawing. Action refers to nongestural actions on manipulatives. The vast majority of cases involved links between two ideas; some involved links among three ideas.

words are presented in plain text. Images of the gestures that occur with each line of transcript are presented below the verbal transcript; the gestures are also numbered (as G1, G2, G3, etc.), and each gesture is described in italic text. Solid arrows are overlaid on the pictures to display relevant motion paths of the hands. Dotted arrows indicate referents of points in cases where those referents are not readily discerned in the video stills. Because the timing of each gesture and its relationship to speech are important to how teachers establish links, we use square brackets to represent the particular words during which each gesture was produced. For cases in which multiple gestures were produced simultaneously, or with temporal overlap, square brackets and curly brackets are used to indicate distinct gestures, if needed. In some cases, teachers produced beat gestures that were superimposed on depictive gestures (i.e., the depictive gesture was moved slightly up and down); such superimposed beats are indicated by underlined text.

Time (Years)	Rabbit population	What was multiplied?	Exponent
0	100	$100 \cdot 1$	
1	180	$100 \cdot 1.8$	$100 \cdot 1.8^1$
2	325	$100 \cdot 1.8 \cdot 1.8$	$100 \cdot 1.8^2$
3	580	$100 \cdot 1.8 \cdot 1.8 \cdot 1.8$	$100 \cdot 1.8^3$
4	1050		

FIGURE 2 Table on the board in the linking episode depicted in Figure 1.

In the example presented in Figure 1, the class is discussing a problem about exponential growth from a worksheet. The problem was as follows: “Given that the growth factor for a rabbit population is 1.8, the starting population was 100, and after four years the population is 1050, how large will the population be in ten years?” There was some initial disagreement among the students regarding the correct answer, and one student suggested the incorrect approach of taking the starting population and multiplying it by the growth factor raised to the power  $10 - 1$  (for an equation of  $y = 100 \cdot 1.8^{(10 - 1)}$ ). The correct equation is actually  $y = 100 \cdot 1.8^x$ , so to find the size of the population in 10 years, one can solve this equation for  $x = 10$ .

Prior to this target link, the teacher in this example had drawn a table on the board; the content of this table is shown in Figure 2. In this linking episode, the teacher links the table entries for number of years (1, 2, 3 etc.) with the table entries for the size of the population at each year (entered in the table as  $100 \cdot 1.8^1$ ,  $100 \cdot 1.8^2$ , etc., in a column labeled “Exponent”). The exponents in these expressions also reflect the number of years (see Figure 2). Thus, the two linked ideas in this episode were (a) the table entries for the number of years and (b) the exponents in the table entries for the size of the population. The teacher states the relationship explicitly in speech, “So this 1 tells me the first year, this 2 tells me the second year. The exponent matches the year.” Along with this speech, the teacher points sequentially to the corresponding information in the table, as seen in Figure 1. She first connects individual values for the first and second years and then links the columns as wholes.

In this example, the teacher uses deictic speech (e.g., “*this* 1”), and her gestures serve to disambiguate the referents of her deictic expressions. In this way, her pointing gestures ground her speech in the physical inscription on the whiteboard, and they may help the students to attend to key relationships among elements of the inscription. The teacher’s final utterance, “The exponent matches the year so we don’t have to do the  $x$  minus one” directly addresses the student misconception that immediately preceded (and perhaps triggered) this linking episode. In this final utterance, the teacher does not use deictic speech but instead uses the full names of the columns (“exponent” and “year”). She also indicates the relevant columns in gestures, even

though her speech alone might have sufficed. Because this link was key to addressing the student's misconception, she presumably wished to communicate it multimodally so that the student would be more likely to take it up appropriately (see Alibali, Nathan, et al., 2013, for further discussion and examples of teachers' gestural responses to *trouble spots*).

The teacher's final gesture, on "x minus one," is a depictive gesture that involves movement across space. Although it is impossible to be certain, it seems plausible that this gesture reflects the *arithmetic is movement along a path* metaphor described by Lakoff and Núñez (2001). Her movement to the right would be viewed as movement to the left by the student whom she is addressing. This would be compatible with subtraction along a conventional number line.

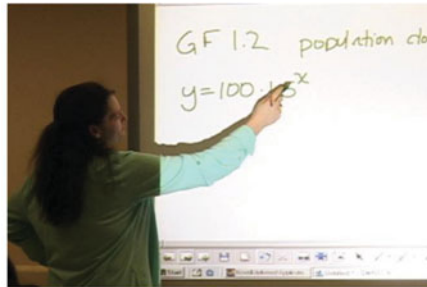
*Links in Which One or Both Ideas Were Expressed in Speech Alone.* In two cases in the dataset (1.2% of target links), both of the linked ideas were expressed in speech alone, and in an additional 14 cases (8.6% of all target links), one of the linked ideas was expressed in speech alone, and the other was expressed multimodally (i.e., in speech and gesture, or in speech, gesture and writing or drawing). In some of these cases, one of the linked ideas was a homework problem that all of the students had completed. In such cases, the teacher often mentioned the homework problem in speech alone and then connected it to a related concept or idea expressed in both gesture and speech.

In other cases where one of the linked ideas was expressed in speech alone, the teacher mentioned or labeled a concept or procedure in speech alone and then described or explained an instantiation of that concept or procedure in both speech and gesture, thus grounding the concept or procedure in its concrete instantiation. For example, in one case, the teacher linked (a) the concept of applying the distributive property multiple times, which he called the "extended distributive property" (expressed in speech alone), and (b) a detailed explanation of an inscription on the board where this concept applied (expressed in both speech and gesture). In another such case, a teacher linked (a) the idea of "simplifying a fraction" (expressed in speech alone) and (b) a detailed explanation of a specific procedure for simplifying a fraction produced with an inscription on the board (expressed in speech and pointing gestures). In these cases, gesture guided students' attention to an instantiation of an abstract idea (i.e., a principle or a procedure) that was mentioned in speech.

*Links in Which One Idea was Expressed in Gesture Alone.* There were two cases in the dataset (1.2% of target links) in which one of the linked ideas was expressed uniquely in gestures without speech at all. One illustrative example occurred in a lesson about exponential growth (from the same unit from which the example presented in Figure 1 was drawn). In this link, presented in Figure 3, the teacher links (a) the exponent ( $x$ ) in the equation  $y = 100 * 1.5^x$  and (b) that same exponent represented in a table that she draws (with columns labeled  $x$  and  $y$ ). At the outset of the linking episode, she points to the exponent in the equation but does not label it as such or say anything at all about it. Thus, this indication of the exponent in the equation occurs in gesture alone (see Figure 4). The teacher then draws the table, fills in 1 for  $x$  (saying, "if  $x$  is 1" while writing the table entry), and asks the students to generate  $y$  (saying, "what would  $y$  be?" while pointing to the relevant, still-empty cell in the  $y$  column of the table). Here her pointing gesture to the exponent is the sole means she uses to refer to the  $x$  in the equation, and she connects it to the  $x$  column in the table, which she expresses in speech and in writing.

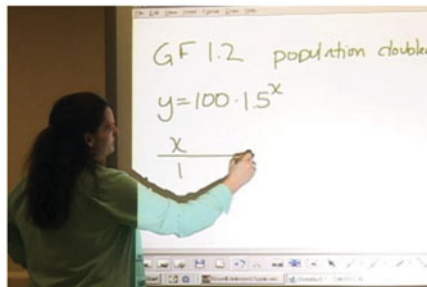
V1. So [ . . . ]

G1. Right hand point to  $x$  as exponent



V2. If [ $x$  is 1],

Draws table where  $x = 1$ .



V3. [What would  $y$ ] be?

G2. Right hand point to  $y$  side of table

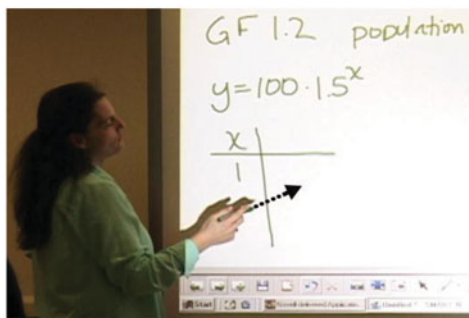


FIGURE 3 Target link in which one of the linked ideas is expressed in gesture alone (i.e., without speech). Square brackets [] identify the words during which each gesture was produced. (Teacher F) (Color figure available online).

In this linking episode, the teacher's pointing gesture connects the equation representation to the tabular representation, thereby giving meaning to the relevant column in the table and perhaps to the exponent in the equation as well. It is noteworthy that the teacher does not refer to the exponent in the equation explicitly in her speech when she points to it (although she does use speech moments later when writing the table entry for the exponent, saying, "if  $x$  is 1"). Her pointing gesture without coexpressive speech seems to be a case of a phenomenon identified by Enfield, Kita, and de Ruiter (2007) who analyzed the pragmatic functions of different types of pointing gestures. Enfield and colleagues noted that at times, speakers produce small, backgrounded pointing gestures in order to respond to "a possible but uncertain lack of referential common ground" between speaker and listener (p. 1,722). Speakers produce such gestures in order to ensure successful reference; however, speakers may choose not to express the gestured information in speech so as not to inappropriately provide information that the listener should already know (termed *over-telling*, which may even offend listeners because of the implication that they do not know something they should). Expressing the relevant information in gesture alone is a subtle way to provide information, which might or might not be needed, without over-telling. Enfield et al. (2007) argue that speakers use such gestures when they are not entirely confident

V1. [That doing a right angle is really easy cause it's like the corner of your paper.]

G1. *Both hands, heels together, depict right angle, 2 superimposed beats*



V2. And if you're knowing that you want something that's bigger than the corner of your paper],

G1. *Gesture held from prior line, 2 additional superimposed beats*



FIGURE 4 Target link involving sequential depictive gestures. Square brackets [] identify the words during which each gesture was produced. Underlined text identifies the words during which superimposed beats were produced. (Teacher C) (Color figure available online) (*Continued*).

V3. [it, it's gonna be more spread out].

G2. *Both hands open wide to represent obtuse angle*



V4. [And if it's smaller than the corner of your paper, it's going to be less than ninety degrees].

G3. *Both hands move closer together to represent acute angle*

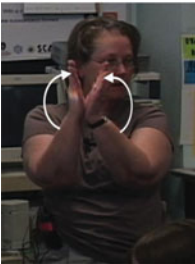


FIGURE 4 (Continued). (Color figure available online).

that their listener will understand their intended referent. In this example, the teacher may not have been certain that her students understood that the table column referred to the exponent, so she expressed that information uniquely in gesture without stating it overtly in speech. Her point provides the students just a bit of extra information, just in case they did not already grasp the link.

*When Gesture was Used to Link, Were Linked Ideas Indicated or Invoked Sequentially or Simultaneously?* For target links in which both (or all) ideas were expressed or referred to in gesture ( $N = 130$ ; 80.2% of all links), teachers gestured to the linked ideas sequentially (i.e., first to one idea and then to the other) in the large majority of cases ( $M = 85\%$ ; range 52–100%). Indeed, every one of the six teachers used sequential gestures in a majority of the links in which they expressed both (or all) linked ideas in speech and gesture.

*Sequential pointing gestures.* A representative example in which a teacher used sequential pointing gestures was presented in Figure 1 above. In discussing the problem about the growth of a population of rabbits, the teacher pointed sequentially to table entries that indicated the number of years and to other table entries that included the exponent in the expression for the total population.

*Sequential Depictive Gestures.* In most cases in which teachers used sequential gestures to connect ideas, they used pointing gestures. However, in some cases teachers used sequential depictive gestures. For example, in one lesson a teacher sought to make connections between (a) a real-world object, namely, the corner of a piece of paper, and various sorts of angles, including (b) a right angle, (c) an obtuse angle, and (d) an acute angle. As seen in Figure 4, the teacher used sequential depictive gestures to represent right, obtuse, and acute angles.

In this example, the teacher expressed her mental simulations of the relevant angles in depictive gestures. These gestures also help ground her mathematical language in familiar visual forms. For example, she depicts a right angle in gesture while saying “right angle,” and she depicts an acute angle in gesture while saying “less than 90 degrees.” Her gestures presumably helped students to mentally simulate the relevant angles as well. The teacher depicted all three angles using both hands in flat palm hand shapes with her wrists together at the vertex (Figure 4). This use of repeated depictive gestures with common elements of form (in this example, common hand shape) is an example of a gestural *catchment*, which McNeill and Duncan (2000) have described as a technique that speakers use to establish cohesion in discourse. By using a similar gestural form to represent all three angles, the teacher conveys that the right, obtuse, and acute angles are all connected, and indeed, they are the “same sort of thing.”

In this linking episode, the teacher also seeks to tie the notion of a right angle to a real-world object with which the students are familiar—the corner of a piece of paper. In doing so, she depicts a right angle in gesture (Figure 4, gesture G1) while introducing the notion of a right angle (saying, “That doing a right angle. . .”), and she then holds this right-angle gesture (rather than putting her hands down) while introducing the comparison to the corner of a piece of paper (saying, “it’s like the corner of your paper”). She also superimposes beat gestures on the right-angle gesture (i.e., she moves the entire right-angle gesture up and down slightly) on the words “like” and “corner” to emphasize those elements of her speech. Thus, her gesture helps to instantiate the connection between a right angle and the corner of a piece of paper by holding the right-angle gesture through the entire verbal comparison and emphasizing “like” and “corner” with superimposed beats.

*Simultaneous Pointing Gestures.* In other links teachers utilized simultaneous gestures to linked representations (e.g., gesture to one idea with one hand and to the other idea with the other hand). Simultaneous gestures were considerably less frequent than sequential gestures, but they were used in at least some links by four of the six teachers. Across teachers, the mean proportion of links involving at least one instance of simultaneous gestures was .15 ( $SD = .18$ ; range .00–.48).

A representative example of a linking episode that includes many simultaneous pointing gestures is presented in Figure 5. In this example, the teacher links (a) the standard notation of a number (206,895) with (b) its corresponding expanded notation ( $2 \cdot 10^5 + 6 \cdot 10^3 + 8 \cdot 10^2 + 9 \cdot 10^1 + 5 \cdot 10^0$ ). He begins by linking  $2 \cdot 10^5$  to the hundred thousands place in 206,895, using sequential gestures: first a trace (with his right index finger) under the  $10^5$  and then a point (with his right index finger) to the 2 in the hundred thousands place, while saying “The ten to the fifth puts it at the hundred thousands.” He holds the point at the 2 in the hundred thousands place, and then points with his left hand back to the  $10^5$ . Thus, he initially gestures to the expanded and standard notations sequentially with his right hand, and then brings in his

V1. [The ten to the fifth] [puts it at the hundred thousands

G1. Right hand trace  $10^5$  from 10 to exponent 5

G2. Right hand point to hundred thousands place



V2. {right?}

G2. Right hand point to hundred thousands place, held from previous line [indicated in square brackets]

G3. Left hand point to  $10^5$  {indicated in curly brackets}



V3. [There's no ten to the fourth {so we have a zero place holder}]

G4. Left hand point to first addition sign between first and second terms [indicated in square brackets]

G5. Right hand point to zero in ten thousands place {indicated in curly brackets}



FIGURE 5 Target link involving sequential and simultaneous points. Square brackets [] identify the words during which each gesture was produced. Curly brackets {} are used to distinguish gestures with temporal overlap. (Teacher A) (Color figure available online) (Continued).

V4. [The six here {is the ten to the third, thousands right};]

G6. Right hand point to 6 in thousands place [indicated in square brackets]

G7. Left hand trace under  $10^3$  {indicated in curly brackets}



V5. [... {...

G8. Left hand begins trace under 8 in  $8 \times 10^2$  [indicated in square brackets]

G9. Right hand point to 8 in hundreds place {indicated in curly brackets}



V6. Eight] hundred}

G8. Left hand continues trace under  $10^2$  (continued from previous line) [indicated in square brackets]

G9. Right hand point to 8 in hundreds place (held from previous line) {indicated in curly brackets}



FIGURE 5 (Continued). (Color figure available online).

V7. [nine{ty}]

G10. Left hand trace under 9 in  $9 \times 10^1$  [indicated in square brackets]

G11. Right hand point to 9 in tens place [indicated in curly brackets]



V8. [five]

G12. Right hand point to 5 in ones place



FIGURE 5 (Continued). (Color figure available online).

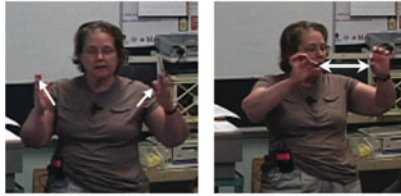
left hand to indicate corresponding parts of the expanded notation and the standard notation at the same time, connected, as it were, through his body. He next uses simultaneous gestures to connect the *absence* of  $10^4$  (which he indicates by pointing to the plus sign between  $2 \cdot 10^5$  and  $6 \cdot 10^3$ , while saying, “there’s no ten to the fourth”) and the ten thousands place (which has a zero place holder), and then in the same way, he proceeds to connect  $10^3$  and the thousands place,  $10^2$  and the hundreds place, and  $10^1$  and the tens place. Thus, in this episode, the teacher delineates the links between the powers of 10 in the expanded notation and the corresponding place value positions in the standard notation.

In this example, the teacher’s simultaneous pointing gestures serve to ground his complex mathematical language in two inscriptions, expanded notation and standard notation, simultaneously. For example, by pointing simultaneously to the relevant portions of both inscriptions, the teacher grounds the term “ten to the fifth” both to its explicit written form in the expanded notation ( $10^5$ ) and to the hundred thousands place in the standard notation. He carries this grounding forward for each element of the expanded notation. In so doing, he both gives meaning to each element in the expanded notation and delineates the relations between the two inscriptions. His gestures link the two inscriptions together element by element.

V1. [well, they're saying that] [that bottom side]

G1. *Both hands held as if holding something (conduit metaphor)*

G2. *Both hands move apart to represent bottom side of triangle*



V2. [on both of those triangles]

G3. *Both hands, each representing a triangle, move down*



V3. [correspond to each other]

G4. *Both hands alternating strokes to represent "correspond"*

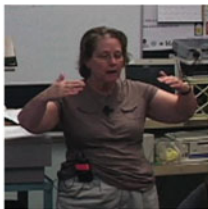


FIGURE 6 Target link involving simultaneous depictive gestures. Square brackets identify the words during which each gesture was produced. Underlined text identifies the words (or parts of words) during which superimposed beats were produced. (Teacher C) (Color figure available online) (*Continued*).

*Simultaneous Depictive Gestures.* Although rare, it was also possible for teachers to communicate links with simultaneous depictive gestures (i.e., one with each hand). For example, in a lesson about similar triangles, one teacher sought to connect (a) the bottom side of one triangle with (b) the bottom side of another similar triangle, by describing an example of two similar right triangles from the students' text. As seen in Figure 6, she used simultaneous depictive gestures to represent the correspondence between the bottom sides of triangle ABC (line segment AC) and triangle DEF (line segment DF).

V4. [what it means by corresponding]

G5. *Both hands held in place, 3 superimposed beats*



V5. [is it means] [that they're both in the same place]

G6. *Both hands move apart to represent bottom side of triangle*

G7. *Both hands, each representing the bottom side of a triangle, 3 superimposed beats*



FIGURE 6 (Continued). (Color figure available online).

In this example, the teacher's gestures express her mental simulation of a figure from the students' textbooks. However, her gestures do not simply trace the image from the text in the air. Instead, they highlight a key element of each of the two triangles—the bottom sides. She first delineates the horizontal bottom side in neutral space (in gestures G2 and G6), and then she represents the two triangles using both hands with flat palm hand shapes (in gestures G3, G4, G5, and G7) in order to highlight that the relevant elements of the triangles in the figure, for present purposes, are their corresponding bottom sides.

The teacher's series of gestures in this linking episode manifest two interwoven catchments—one that captures the general notion of “bottom side,” which she depicts using both hands moving apart in neutral space (in G2 and G6), and one that captures the two triangles' bottom sides, which she depicts with simultaneous depictive gestures with a palm down hand shape (in G3, G4, G5, and G7). In gesture G4, the teacher also layers on the additional notion of “correspondence” by producing an alternating motion with her hands in the palm-down two-triangles configuration. Thus, her gestures highlight the correspondence between the bottom sides of the two triangles in a number of ways. Her gestures presumably help call to mind the textbook figure of the two triangles for the students, and they presumably also help students to focus on the critical aspect for the present moment, the bottom sides.

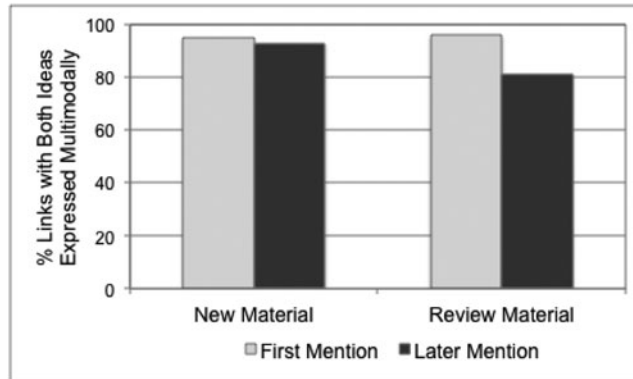


FIGURE 7 Percent of links in which both ideas were expressed multimodally, as a function of whether the material was new or review, and whether it was mentioned for the first time or for a later time within the lesson.

### Did Teachers' Expression of Links Vary Depending on Whether the Material Was New or Review?

Thus far, we have established that teachers use a variety of types of links in their instruction. Links vary in terms of whether both of the linked ideas are expressed multimodally or at least one of the linked ideas is expressed in speech alone. For links in which both linked ideas are expressed multimodally, links also vary in terms of whether both ideas are referred to sequentially or simultaneously. What factors might give rise to this variation?

Based on prior research (Alibali & Nathan, 2007), we hypothesized that teachers would express links in multiple modalities (e.g., in speech and gesture together, or in speech and writing together) more often for material that was new to the students than for material that was review. In addressing this question, we also examined whether the material was expressed for the first time within the lesson or whether it had previously been mentioned within that lesson.

The data are presented in Figure 7. When mentioned for the first time within the lesson, links that involved new material and links that involved review material were equally likely to be expressed multimodally (95% and 96% of such links, respectively). However, review material that was mentioned for the second (or later) time within the lesson was considerably less likely to be expressed multimodally (81% of such links).

We also examined teachers' gesture rates in target links as a function of whether those links were new or review. For each target link, we summed the number of substantive gestures (i.e., gestures of all types except beats), divided by the number of words, and multiplied by 100 to obtain the gesture rate per 100 words (a measure commonly used in the gesture literature). We then compared gesture rates for links about new material and links about review material for each teacher (one teacher was excluded from this analysis because all of the links she produced during the lessons were about review material, so there was no basis for comparing new to review material). As expected, teachers gestured at a higher rate for links that involved new material ( $M = 20.1$  vs.  $14.9$ ), paired  $t(4) = 2.26$ ,  $p = .04$ , one-tailed. Thus, teachers were more likely to use

gestures along with speech when communicating about material that was likely to be unfamiliar and therefore more challenging for their students.

## DISCUSSION

### Teachers Make Connections Multimodally

Making connections is an essential part of mathematics instruction. The middle school lessons that we examined in this research were rich with connections—and in the vast majority of cases, teachers communicated about these connections multimodally. Our findings align with those of other studies showing that gestures play an integral role in teachers' instructional communication (e.g., Flevares & Perry, 2001). The present findings extend past work to focus on middle school mathematics and to focus specifically on connection making.

Our data indicate that teachers make links multimodally most consistently when the content of the link is mentioned for the first time within a lesson, regardless of whether that content is new or review. Teachers are less likely to make links multimodally when those links concern familiar material that has been mentioned previously within the lesson. Thus, teachers appear to communicate multimodally when students need it most—they are sensitive to the state of students' knowledge in their communication about links.

One interpretation of this finding is that gesture, and multimodal communication more generally, ensures or promotes common ground in the classroom, manifested as intersubjectivity between teacher and students. Nathan and Alibali (2011) identified two types of circumstances in which gestures play this role in fostering intersubjectivity. In one set of circumstances, teachers use gesture-rich communication to avoid anticipated breakdowns in comprehension or to institute conversational repair—that is, to address potential or actual trouble spots in instruction (see Alibali, Nathan, et al., 2013). In a second set of circumstances, teachers recognize that when students encounter a novel idea, they may lack sufficient common ground with the teacher to competently use the new idea. In these circumstances, teachers use gestures to help students give meaning to the new idea and to help them use it properly.

Even for review material that had been mentioned previously, teachers in this study used multiple modalities more often than not. In this regard, the overall rate of substantive gestures (i.e., including depictive, pointing, and writing gestures, but excluding beats and writing performed as a functional act) that we observed in target links in this research was substantially higher than the gesture rates that have been reported in studies of other discourse contexts, such as narrative retell. In our corpus, teachers' average gesture rate in target links was 16.9 substantive gestures per 100 words; for comparison, Alibali, Heath, and Myers (2001) reported that adults retelling a cartoon story to a naive listener produced just under 10 such gestures per 100 words. It may be the case that communication during instruction is heavily multimodal across the board. Indeed, this is just as might be expected in light of recent evidence that speakers gesture more when the stakes of communication are high (Kelly, Byrne, & Holler, 2011). Teachers are highly motivated to communicate effectively, so instruction is presumably a high stakes communicative situation.

## Teachers Use Gestures to Express Relatedness in Several Ways

In our data, teachers expressed relatedness of ideas in various ways. In some cases, relatedness was expressed via gestural catchments—sets of gestures that share formal characteristics, such as hand shape or motion trajectory (McNeill & Duncan, 2000). In other cases, relatedness of ideas was expressed via corresponding pointing gestures—also a form of gestural catchment, albeit one that may not be so obvious because points are such a common gestural form. As we have documented, in many cases teachers use gestures and speech to delineate the multiple correspondences that exist between elements of mathematical representations (e.g., individual table entries in Figure 3, parts of numerical representations in Figure 5). Our findings align with other reports that highlight teachers' use of gesture as a means for establishing cohesion among ideas and representations in instructional settings (Nathan & Alibali, 2011; Nathan et al., 2013).

Gesture is adept at expressing correspondences among ideas, at least in part because humans can indicate or represent more than one thing at the same time using their two hands. Simultaneous gestures to linked ideas were not infrequent in our data set, and we suspect that such gestures may play an important role in students' comprehension of connections. When teachers use simultaneous gestures to linked ideas, they refer to both linked ideas and actually embody the linking relationship between those ideas all at once. In so doing, they may foster attention to both of the related ideas and their relationship at the same time. A number of studies have documented the value of simultaneous presentation of related information for learning. For example, infants are more likely to learn categories if multiple exemplars are presented simultaneously rather than sequentially (e.g., Oakes & Kovack-Lesh, 2007; Oakes & Ribar, 2005), and preschoolers are more likely to abstract relationships from examples when those examples are presented simultaneously (Christie & Gentner, 2010). Indeed, the effectiveness of a *contrasting cases* approach in instruction hinges on students' comparing multiple, simultaneously presented examples (Hattikudur & Alibali, 2010; Rittle-Johnson & Star, 2007; 2009; Star & Rittle-Johnson, 2009).

In light of this evidence for the value of simultaneous presentation, it seems possible that using simultaneous gestures to link related ideas could foster greater learning than sequential gestures. We obtained some preliminary evidence on this possibility in a recent study. We videotaped a teacher instructing small groups of students both before and after a professional development tutorial in which we encouraged the teacher to use simultaneous gestures to link related ideas (Alibali, Young, et al., 2013). The lessons focused on slope and intercept and how they are represented in Cartesian graphs and algebraic equations. The videotaped lessons were then presented to a new set of middle school students.

The teacher altered his gestures after the professional development tutorial, demonstrating that teachers can indeed vary their gestures when instructed to do so (see also Hostetter, Bieda, Alibali, Nathan, & Knuth, 2006). In particular, he produced many more simultaneous linking gestures despite producing a comparable number of linking episodes overall. Moreover, students who viewed the gesture-enhanced video lesson performed better on a posttest than did students who watched the baseline video. Thus, students learned more from the lesson that incorporated more simultaneous linking gestures. Of course, this was a small-scale study with video instruction from only a single teacher; however, the findings are suggestive. Based on these preliminary findings, we hypothesize that students learn more from linking episodes in which teachers' gestures foster simultaneous attention to the linked ideas. The direct manifestation of connections that are

literally embodied by simultaneous linking gestures may help to reduce the cognitive demands of attending to and integrating across multiple ideas, as compared to the temporal unfolding of sequential linking gestures.

Further work is needed to empirically establish the value of simultaneous as opposed to sequential linking gestures. However, it seems likely that the benefits of simultaneous linking may depend on students' level of prior knowledge, as is the case for contrasting cases instruction (Rittle-Johnson, Star, & Durkin, 2009). In our corpus, there was substantial variability across teachers in their use of simultaneous gestures in target links. If such gestures prove to be beneficial for student learning, this could present an opportunity for teachers to learn to use gesture in pedagogically effective ways.

### Implications for Teacher Practice and Professional Development

As the foregoing discussion implies, scientific knowledge about the impact of teachers' gestures on students' learning is not yet at a point where definitive, empirically based recommendations about practice can be made. However, the present data do have some tentative implications for teacher practice and professional development. In this regard, we wish to emphasize that the pedagogical implications we draw are contingent on the specific connections being communicated having pedagogical value. Effective communication of ill-advised or inaccurate connections is sure to be detrimental to student learning. In the remarks that follow, we assume that the connections in question are pedagogically valuable ones.

One implication has to do with teachers' knowledge about the importance of communicating key ideas and links among them multimodally. A large body of evidence indicates that information that is communicated multimodally is more likely to be taken up by listeners than information that is communicated in speech alone. A meta-analytic review of this literature, which included studies in both conversational and instructional settings, provided strong evidence that gestures are beneficial for comprehension (Hostetter, 2011). In light of this evidence, our current findings suggest that teachers do not always use gestures in ways that might best foster students' comprehension. In roughly 10% of the linking episodes we observed, one or both of the linked ideas were not expressed multimodally. Linking episodes in which some ideas are expressed in a single modality—either speech alone or gesture alone—may represent potential missed opportunities for scaffolding students' comprehension and learning about links between ideas. Thus, professional development experiences that highlight the value of multimodal communication may prove to be valuable.

A second implication has to do with teachers' awareness of the range of ways in which gesture can be used to communicate links between ideas. Our data revealed substantial individual differences among the teachers in our sample in their use of gesture to link ideas. Some teachers used a range of gestural methods for communicating links, and other teachers used relatively few. Future research is needed to evaluate whether variations in teachers' gestures are associated with variations in learning on the part of students. It may be the case that certain ways of gesturing about links between ideas are especially beneficial for student learning.

In light of these considerations, we suggest that teachers may find value in professional development experiences that highlight ways to communicate relationships via gesture. Gestures are readily available to all teachers, and they are easily customizable for any communicative

situation. Teachers may find it worthwhile to learn about different ways to effectively use gestures in communicating connections that they deem pedagogically valuable for their students. Moreover, this may hold true especially for teachers who do not spontaneously rely on gestures in their communication, that is, teachers who ordinarily gesture relatively infrequently.

In sum, there is still much work to be done before the present findings can be translated into recommendations that can be implemented in teacher professional development. We acknowledge that effective professional development on how connections are communicated requires a solid foundation of understanding what sorts of connections are valuable for students. Nevertheless, we believe that, at a minimum, the present work highlights the potential value of attending to gesture when considering instructional communication, especially communication about connections. This lens may be of interest and value to both practicing and future teachers.

### Implications for Theories of Gesture Production and Comprehension

Several aspects of the current findings also have implications for contemporary theories of gesture production and comprehension. In his seminal work on gesture in narrative, David McNeill (1992) argued that spontaneous gestures express the information within an utterance that has the greatest *communicative dynamism*—that is, the information that is most novel within the current context and that “pushes the communication forward” to the greatest extent (p. 207). The present findings suggest that this claim holds true not only at the level of the individual utterance but also at higher levels of discourse structure. We coded our data at the level of the target link—which often stretched over multiple lines of transcript—and found that teachers produced gestures at higher rates when talking about links that were new and when talking about specific links for the first time within a given lesson. These data provide empirical support for McNeill’s claims about the association of gesture with information that is novel within the current context, and they suggest that this association holds true at levels of discourse structure broader than the individual utterance.

The present findings also highlight the value of examining catchments in speakers’ gestural behavior. Teachers in this study regularly used gestural catchments when speaking about related ideas. McNeill and colleagues (McNeill et al., 2001) have argued that such catchments occur because recurrent imagery in a speaker’s thinking is manifested in recurrent features in the speaker’s gestures. Our findings are compatible with this view, and they also raise the question of whether speakers use catchments strategically to purposefully communicate relatedness. We suggest that when the relatedness of information is itself the speaking topic, speakers may strategically use catchments in order to insure that this relatedness is communicated to their listeners. Future experimental studies could be designed to investigate whether such catchments make a difference for listeners’ comprehension of connections between ideas.

The present findings also highlight several other important questions for future work. One set of questions has to do with the communicative significance of sequential versus simultaneous gestures. Does gesturing about related ideas simultaneously as opposed to sequentially make a difference for listeners’ understanding? Another set of questions has to do with individual differences in gesture production. Why do some teachers gesture more than others, and are variations in gesture content associated with systematic variations in speech content or quality? Many questions about gesture production and comprehension remain unanswered, and the present

study underscores that, in some cases, the answers may have practical relevance for instructional communication and learning.

## CONCLUSION

In sum, in this study we have documented how teachers express links between ideas in speech, gestures, and other modalities in naturalistic classroom instruction in middle school mathematics. We have shown that when teachers communicate about links between ideas, they nearly always communicate using multiple modalities. Moreover, teachers gestured at a higher rate when communicating links that were new for the students than when communicating links that were review. On the whole, our findings support the idea that teachers utilize depictive and pointing gestures along with other visual and verbal representations as an integral part of their efforts to communicate during mathematics instruction. These findings pave the way for future studies about how to make teachers' instructional communication optimally effective.

## ACKNOWLEDGMENTS

We thank Maia Ledesma and Kristen Bieda for their contributions to this research, and we thank the participating teachers for opening their classrooms to us and for agreeing to be videotaped.

## FUNDING

This research was supported by grant # R305H060097 from the U. S. Department of Education, Institute of Education Sciences (Martha W. Alibali, PI). All opinions expressed herein are those of the authors and not the U. S. Department of Education.

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## APPENDIX: POSTLESSON INTERVIEW TOPICS AND QUESTIONS

### Questions About the Lesson

What material in this lesson was new and what was review for the students?

What were the parts of this lesson?

Was there anything that you were surprised by in the students' reactions to the lesson?

Did you notice the students having difficulty with any particular part of the lesson? What in particular drew your attention to the fact that particular students were not following or understanding the lesson?

### Questions About Representations

I noticed you wrote (x) on the board (transparency, etc.) and spent some time discussing it; what was your goal in setting up the representation in that way?

I noticed you made a link between (x) and (y) on the board (transparency, etc.); what was your goal in linking those two representations in that way?

How did you set up or link the representations on the board or transparency?

### Observation-Based Questions (About Anything of Note That Occurred During the Lesson)

I saw that you did (x) . . . Why?